Name: _____

- You have fifty minutes to complete this mock exam.

1. Evaluate the following derivatives.

   (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\cos x\right)$

   (b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\csc x\right)$

   (c) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\tan x\right)$

   (d) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1} x\right)$

   (e) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(5^x\right)$

2. Differentiate the function $y = \dfrac{1 - xe^x}{x + e^x}$.

3. Differentiate the function $f(\theta) = \sin(\tan(2\theta))$.

4. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the curve $y \sin(x^2) = x \sin(y^2)$.

5. Differentiate the function $f(x) = x^{\cos x}$.

6. A curve has the property that at each point, the slope of its tangent line is half the y-coordinate of that point. If the curve passes through the point $(1, -3)$, find an equation for the curve.

7. If a ball is thrown vertically upwards with a velocity of 80 feet per second, its height after t seconds is $h = 80t - 16t^2$. What is the maximum height of the ball?

8. At noon, ship A is 100 kilometers west of ship B. Ship A is sailing south at 35 kilometers per hour, and ship B is sailing north at 25 kilometers per hour. How fast is the distance between the ships changing at 4:00 PM?

9. A piece of wire 10 meters long is cut into two pieces. One piece is bent into a square and the other into an equilateral triangle. How should the wire be cut so that the total area enclosed by the square and triangle is maximized?

10. Consider the function $f(x) = x^5 - 2x^3 + x$.

    (a) Find the intervals on which f is increasing and decreasing.

    (b) Find the local maximum and minimum values.

    (c) Find the intervals of concavity and inflection points.

    (d) Use the information from parts (a)-(c) to sketch the graph of f.

11. Evaluate the following limits.

    (a) $\displaystyle\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$

    (b) $\displaystyle\lim_{x \to 0^+} x \ln x$

    (c) $\displaystyle\lim_{x \to \infty} \frac{x^{3,000}}{e^{0.1x}}$