MATH 231 / Spring 2010

Disclaimer: This review was written by your TA, Ben Reiniger. The final exam will be written by Dr. Kerman. As such, these problems should not be taken as a complete representative of the actual final; rather, they are meant to be a supplement to your review of previous homework, quiz, and exam experiences.

1 Integrals

1. $\int e^{x+e^x}dx \quad = e^{e^x} + C$

2. $\int \cos x(1+\sin^2 x)dx \quad = \sin x + \frac{\sin x}{3} + C$

3. $\int \frac{e^{\tan^{-1} y}}{1+y^2}dy \quad = e^{\tan^{-1} y} + C$

4. $\int_{-2}^{2} |x^2 - 4x| \; dx \quad = 16$

5. $\int_{-\pi/4}^{\pi/4} \frac{\sec\theta\tan\theta}{\sec^2\theta - \sec\theta}d\theta$ DNE

6. $\int (1+\sqrt{x})^8 dx \quad = \frac{(1+\sqrt{x})^{10}}{5} - \frac{2(1+\sqrt{x})^9}{9} + C$

7. $\int \theta\tan^2\theta \; d\theta \quad = -\frac{1}{2}\theta^2 + \theta\tan\theta - \ln(\cos\theta) + C$

8. $\int_2^5 \frac{8x^2}{(x-1)^2(x^2+1)^2}dx \quad = \frac{153}{130} - 2\tan^{-1} 5 + 2\tan^{-1} 2$

9. $\int \frac{1}{\sqrt{x-2}+\sqrt{x}}dx \quad = \frac{x^{3/2}}{3} - \frac{(x-2)^{3/2}}{3} + C$

10. $\int_{-\infty}^{0} \frac{3^x + 5^x}{2^x}dx \quad = \frac{1}{\ln(3/2)} + \frac{1}{\ln(5/2)}$

11. $\int \frac{1+2x+\tan^{-1} x}{1+x^2}dx \quad = \tan^{-1} x + \ln(1+x^2) + \frac{1}{2}(\tan^{-1} x)^2 + C$

12. $\int (\sin x + \cos x)^2 dx \quad = x + \sin^2 x + C$

13. $\int_2^{\infty} \frac{dx}{x\ln x} \quad = +\infty$

14. $\int \ln(x^2+1)dx \quad = x\ln(x^2+1) - 2x + 2\tan^{-1} x + C$

15. $\int \frac{dx}{x^5\sqrt{4x^2-1}} \quad = 6\sec^{-1} x + \frac{3\sqrt{4x^2-1}}{2x^2} + \frac{\sqrt{4x^2-1}}{4x^4} + C$

2 Series

1. Determine the intervals of convergence for the following series:

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)}{(5n)!}(2x-1)^n$ $\mathbb{R}$

   (b) $\displaystyle\sum_{n=18}^{\infty} n!(2x-1)^n$ $\left\{\frac{1}{2}\right\}$

   (c) $\displaystyle\sum_{n=2}^{\infty} 2^n x^n$ $\left(-\frac{1}{2}, \frac{1}{2}\right)$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$ $[-1, 1)$

2. Determine the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \cdot n! \cdot 5 \cdot 10 \cdot 15 \cdot \ldots \cdot (5n)}{3^n (2n)!}(2x-1)^n$ radius $= \frac{6}{5}$

   (You can also test the endpoints, but it's tricky. Consider the ratio of consecutive terms, but don't take the limit. Once you've done this, what can you say about how the terms change?)
   interval is $\left(-\frac{7}{10}, \frac{22}{5}\right)$

3. Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n! - 3^n}$ converges. It does.

4. Let $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n^2 + 3n + 2}$. Determine whether the following exist, and find them exactly if possible:
   $f(-1)$ exists, $f(0)$ $= 1/2$, $f(1)$ $= 1$, and $f(2)$ DNE.

5. Let $g(x) = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{e^{2n+1}}$. Determine whether the following exist, and find them exactly if possible:
   $g(-10)$ DNE, $g(0)$ $= \frac{1}{e}$, $g(1)$ $= \frac{e}{e^2-1}$, and $g(e^2)$ DNE.

6. Let $h(x) = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$. Determine whether the following exist, and find them exactly if possible:
   $h(-1)$ $= 1/e$, $h(0)$ $= 1$, and $h(2)$ $= e^2$.

7. Let $y(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$. Determine whether the following exist, and find them exactly if possible:
   $y(-\pi/2)$ exists, $y(0)$ $= 1$, and $y(1)$ $= \cos(1)$.

3 Parametric Equations and Polar Coordinates

1. Consider the parametric curve $x = \sin t$, $y = \sec t$. Find a Cartesian equation for this curve. Does the parametric curve trace out the entire Cartesian curve once?
   $x^2 + \frac{1}{y^2} = 1$ The parametric curve traces out the entire Cartesian one, but does so infinitely often (periodically, with period 2π)

2. Consider the parametric curve $x = \tan t$, $y = \sec t$. Find a Cartesian equation for this curve. This type of curve has a name; what is it? Does the parametric curve trace out the entire Cartesian one once? Find where the tangent line to this curve is horizontal or vertical. Find an equation for the tangent line to the curve at the point $(\sqrt{3}, 2)$.
   $-x^2 + y^2 = 1$ This is a hyperbola. Again, the curve is traced out infinitely often, with period 2π.

We have horizontal asymptotes at $(x, y) = (0, \pm 1)$, and no vertical asymptotes. At $(\sqrt{3}, 2)$ the tangent line is given by $y - 2 = \frac{\sqrt{3}}{2}(x - \sqrt{3})$.

3. Consider the parametric curve $x = 3 \sin t$, $y = 4 \sin t$.
   (a) Calculate its arc length using the parametric formula for arc length. 10
   (b) Calculate its arc length by first transforming into a Cartesian equation. (Is the entire Cartesian curve traced out once?) 10; it is a line segment

4. Find the length of the loop of the parametric curve $x = 3t - t^3$, $y = 3t^2$. $24\sqrt{3}$

5. Find the area enclosed by the astroid with parametric equations $x = \cos^3 t$, $y = \sin^3 t$. $\frac{3\pi}{8}$

6. Consider the polar curve $r = \sin \theta$. Find a Cartesian equation for this curve. This type of curve has a name; what is it?
   $y = x^2 + y^2$ This is a circle.

7. Consider the polar curve $r = \sin 3\theta$. It is a three-leaf rose. Find the area of one leaf. $\frac{\pi}{12}$

8. Consider the polar curve $r = 2 - 2 \cos \theta$. Find its arc length. 16