Workshop 24 December 1, 2011

1. Suppose f is an even function. What does the polar curve $r = f(\theta)$ look like? What if f is an odd function?

2. Consider the polar curve $r = \sin(n\theta)$, where n is an integer.

   (a) Plot the curves for each n between 1 and 7.

   (b) How does the number of leaves in the curve depend on n? Why?

   (c) Assuming your conjecture is correct, compute the total area enclosed by the curve. How much does your answer depend on n?

   (d) What happens if we use instead the curve $r = |\sin(n\theta)|$?

3. Plot the polar curve $r = \theta^2$. Find the arc length of this curve from $\theta = 0$ to $\theta = 2\pi$.

4. Find the surface area of the surface obtained by rotating the lemniscate $r^2 = \cos(2\theta)$ about the line $\theta = 0$.

5. The following will demonstrate an interesting property of the curve $y = 1/x$.

   (a) First, find a polar representation of this curve.

   (b) We want to find the area bounded between the curve $y = 1/x$, $y = mx$, and $y = 2mx$ for a positive constant m. First sketch the situation when $m = 1$ and when $m = 2$. Which of these two areas looks larger?

   (c) Now we'll integrate to find the area precisely. Explain why this would be difficult in rectangular coordinates. Then convert everything to polar and find the area. When this is done, which area was really larger, when $m = 1$ or when $m = 2$? What happens with other positive m?

6. Find the Cartesian equation of the tangent line to the polar curve $r = 2 - \sin\theta$ at $\theta = \pi/3$.

7. Plot the polar curve $r = \theta$. Find the arc length of this curve from $\theta = 0$ to $\theta = 2\pi$. (We've seen the resulting integral before, but it isn't pleasant...)