# Workshop 22 November 17, 2011

1. *substitutions and parametrizations*
   (a) Remember integrals? Set up an integral in the old way to get the area of a half-disk with radius 1.
   (b) How can you evaluate the resulting integral? (Note: you should change the bounds of the integral to match your new variable.)
   (c) Give a parametrization of the boundary of the region from 1a (a half-circle). Use this parametrization and your new technique for finding areas bounded by parametric curves to set up an integral representing the area.
   (d) How does your work on 1b compare to your work on 1c?

2. *Let's unwind a little.* A string is coiled tightly around a circle of radius $r$ centered at the origin. Now we start to unwind it (while always keeping the string taut). The end of the string traces out a curve called the *involute* of the circle. Find a parametrization of such a curve. (Hint: since the string is kept taut, the bit of string that's been unwound is always tangent to the circle. Use the angle $\theta$ between the $x$-axis and the point of tangency as your parameter. You'll need a good geometric diagram.)

3. *Moo.* A cow is tied to circular silo with radius $r$ by a rope just long enough to reach the opposite side of the silo. Compute the area of grass available to the cow. (Hint: use problem (2) for part of the area, and simple geometry for the rest.)

4. *Bézier curves*

   Bézier curves are very useful in computer-aided design. Let's explore the class of these curves that have only three *control points*, say $P_0 = (x_0, y_0)$, $P_1 = (x_1, y_1)$, and $P_2 = (x_2, y_2)$. We define the curve then to be given by

   $$x = x_0(1-t)^2 + x_1 t(1-t) + x_2 t^2,$$
   $$y = y_0(1-t)^2 + y_1 t(1-t) + y_2 t^2.$$

   (a) What point do you get when $t = 0$? When $t = 1$?
   (b) Do you get $P_1$ when $t = 1/2$?
   (c) What is the equation for the line segment joining $P_0$ to $P_1$? What about for the line segment joining $P_1$ to $P_2$?
   (d) Find the slope of the tangent line to the curve at $t = 0$. What do you notice? Do the same at $t = 1$.