Workshop 21 November 15, 2011

1. A common probability distribution is the *Poisson distribution.* It is a good approximation to almost anything random that can be counted, for example the number of webpage hits during a day. This distribution is discrete but not finite: the possible outcomes are all nonnegative integers. Specifically, the probability of having exactly k occurrences is given by $\dfrac{\lambda^k e^{-\lambda}}{k!}$, where λ is a parameter depending on the specific situation.

   (a) Show that this is really a probability distribution, i.e. show that $\displaystyle\sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} = 1$.

   (b) Find the expected number of occurrences by computing $\displaystyle\sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} \cdot k$.

2. Find a parametrization of the line segment going from $(1,5)$ to $(-3,2)$. Make sure that you're going in the right direction! What is the domain of your parameter? More generally, find a parametrization of the line segment going from $A = (a_1, a_2)$ to $B = (b_1, b_2)$. Again, make sure the path goes in the right direction, and specify the domain of the parameter.

3. Let $r(t) = (\tan t, \tan^2 t)$ for $t \in [\pi/4, 3\pi/4]$ (I'm cheating a little bit here). Eliminate the parameter to find a cartesian equation of a curve that contains the image of this function. Describe how this parametrization traces out its image (what is its image anyway?).

4. Consider $r_1(t) = (\cos t, \sin t)$ for $t \in [0, 2\pi]$. What curve does it trace out? What is $r_1(\pi/3)$? What are $\left.\dfrac{dx}{dt}\right|_{t=\pi/3}$ and $\left.\dfrac{dy}{dt}\right|_{t=\pi/3}$? Now consider $r_2(t) = (\sin t, \cos t)$ for the same t range. What curve does it trace out, and what is $r_2(\pi/6)$? What are the derivatives at $t = \pi/6$? Explain the difference by considering the parametrizations as movement of a particle. What is the slope of the tangent line to the curve at the point $(1/2, \sqrt{3}/2)$? Do the results you get from using r_1 and r_2 agree on the slope of the tangent line?

5. *substitutions and parametrizations*

   (a) Remember integrals? Set up an integral in the old way to get the area of a half-disk with radius 1.

   (b) How can you evaluate the resulting integral? (Note: you should change the bounds of the integral to match your new variable.)

   (c) Give a parametrization of the boundary of the region from 5a (a half-circle). Use this parametrization and your new technique for finding areas bounded by parametric curves to set up an integral representing the area.

   (d) How does your work on 5b compare to your work on 5c?

6. *Bézier curves*

Bézier curves are a generalization of the kind of parametric curve described in problem (2), and are very useful in computer-aided design. Let's explore the class of these curves that have three *control points*, say $P_0 = (x_0, y_0)$, $P_1 = (x_1, y_1)$, and $P_2 = (x_2, y_2)$. We define the curve then to be given by

$$x = x_0(1-t)^2 + x_1 t(1-t) + x_2 t^2,$$
$$y = y_0(1-t)^2 + y_1 t(1-t) + y_2 t^2.$$

(a) What point do you get when $t = 0$? When $t = 1$?

(b) Do you get P_1 when $t = 1/2$?

(c) What is the equation for the line segment joining P_0 to P_1? What about for the line segment joining P_1 to P_2?

(d) Find the slope of the tangent line to the curve at $t = 0$. What do you notice? Do the same at $t = 1$.