Workshop 9 September 27, 2011

Dam Pirate Symmetry

1. Suppose a dam holds back an average height of 30m of water, and as the dam supervisor you have memorized the amount of force put on the dam when the water is at this height. After a particularly wet season, the water is up to 40m. Can you calculate the new force of the water by just adding the force on the upper 10m of the dam to your memorized number? Why or why not?

2. Pirate Joe has a barrel filled with stuff that he doesn't want found. He decides to tether the barrel underwater. The sides of the barrel are pretty sturdy, but the top and bottom might cave in with enough pressure; specifically, the top can withstand 110835N of force, and the bottom can withstand 135835N. The barrel is 880mm tall and the top and bottom have diameter 610mm. Recall that the density of water is 1000kg/m^3, and that gravity's acceleration is approximately 9.8m/s^2 (you may use a calculator to do the arithmetic.) Assume that the contents of the barrel don't exert significant force.

   (a) Suppose Pirate Joe tethers the barrel so that it floats right-side-up. What is the maximum depth at which he can safely keep the barrel? (Hint: this is easy!)

   (b) Suppose Pirate Joe tethers the barrel so that it floats on its side. What is the maximum depth to which he can safely keep the barrel? (Hints: It may help to set the origin as the center of the barrel; this way the integrals needed will be symmetric. Then keep your eyes out for integrals that give geometric answers.)

   (c) Which way should Pirate Joe submerge his barrel so as to maximize the depth of the shallowest part of the barrel (and hence its difficulty in being discovered)? What is this depth?

3. You may have noticed something in the last problem that's worth pointing out. Suppose you have a flat surface that is submerged (completely) underwater in such a way that there is a horizontal axis of symmetry to this surface. Now rotate the surface so that it lies flat (the center of the surface should remain at the same depth). Prove that the hydrostatic force on the surface is the same in these two situations.