Workshop 8 September 22, 2011

This worksheet is filled with opportunities for pretty pictures! Take advantage of them and use the chalkboard!

1. Improper integrals, though defined in the only way that seems natural, allow for some pretty unnatural things... Gabriel's Horn is the surface obtained by revolving the curve $y = 1/x$ for $x \geq 1$ about the x-axis. It's also pictured in your text.

   (a) Show that the area under the curve $y = 1/x$ for $x \geq 1$ is *infinite*.

   (b) Show that the volume enclosed by Gabriels Horn is *finite*.

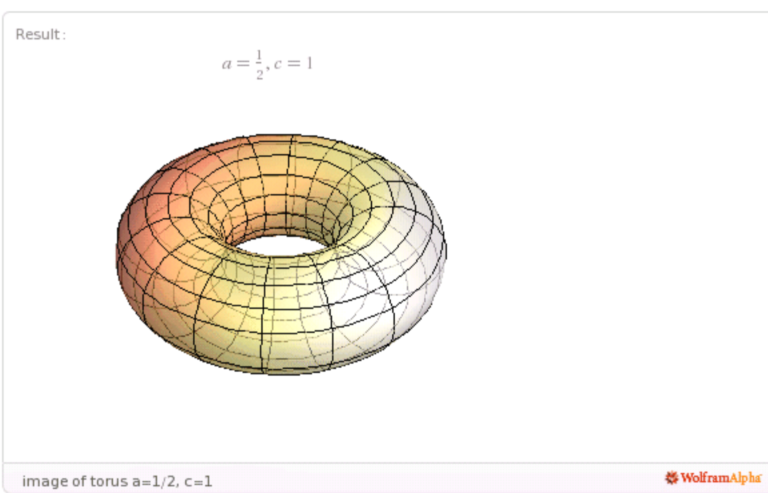
   (c) Show that the surface area of Gabriel's Horn is *infinite*. (Hint: You can, but shouldn't, explicitly compute an antiderivative; instead, use a comparison.)

   The first and second results are surprising; the area is infinite, but when you rotate it around an axis the resulting volume is finite! The second and third results are also weird; you can fill the horn with paint but can't paint the surface!

2. You can think of arc length (or even surface area) integrals as Riemann sums, just like any other integral. Consider the function $y = x^2$ on the interval $x \in [0, 1]$. Break the interval into 4 subintervals, then approximate the curve by straight-line segments. Find the length of each segment, and add them together to get an approximation of the curves arc length. What happens if you take more intervals? Can you say whether these approximations are under- or over-estimates? Why? Does it depend on the function?

3. Set up an integral representing the arc length of one full period of the curve $y = \cos x$. Use symmetry and a very basic approximation technique to approximate the value. If your approximation technique allows it, say whether your approximation is an over- or under-estimate.

4. A *torus* is a mathematical object like the one shown below. One way to get such a torus is to start with a circle in the plane, centered say at $(c, 0)$ with radius r (to really be a torus, we need $r < c$). Then rotate this circle about the y-axis.



Compute the surface area of a torus (in terms of c and r). (Hint: this will be easier if you use a bit of symmetry.) If we let $c = 0$, what shape do you get, and what area does your formula produce?

5. You'll need a good calculator for this problem. If you can program it, that would be especially handy.

   (a) Plot the curve $y = x$ on the interval $x \in [0, 1]$. Compute its arc length.

   (b) Do the same for $y = x^2$.

   (c) What about $y = x^3$?

   (d) Try $y = x^n$ for some larger integer n (or program your calculator to give a list of several arc lengths and corresponding graphs).

   (e) What do you think happens as $n \to \infty$?

   Note: this sequence of functions is a very common example in analysis courses (for reasons other than the arc length).