Workshop 6 September 13, 2011

1. *Simpson's rule*
   (a) How many points determine a quadratic polynomial?
   (b) If you have three equally-spaced x-values, say x_0, $x_1 = x_0 + h$, and $x_2 = x_0 + 2h$, and you know the values of a quadratic function f at those three points, then you could theoretically figure out what f is exactly. (That's what your answer to 1a means.) It would be a large-ish mess of systems of equations, but you could do it. A nice feature of this process though is that you know exactly what the area under this graph is: it is $\frac{h}{3}\left(1f(x_0) + 4f(x_1) + 1f(x_2)\right)$. Use this to derive the formula in Simpson's rule.

2. You learned in Calculus 1 that the integral is defined as a limit of Riemann sums, i.e. that the integral is estimated by finite Riemann sums. Using left and right endpoint sums give you a couple of estimates, but other methods give better estimates. Draw five graphs of $y = x^2$ for x between 0 and 6 say. Draw in the areas given by left endpoint, right endpoint, midpoint, trapezoidal, and Simpson's estimations with 6 subintervals (one estimation drawing per graph). Are these estimates exact, over-, or under-estimates? Why? Write down, but do not evaluate (unless you have a calculator and wish to) an expression that represents these estimates. Finally (hopefully the easy part), compute the exact value. (You have two ways to do this now!)

3. Find the area under the curve $y = \dfrac{x^2+1}{3x-x^2}$ between $x = 1$ and $x = 2$.

4. Make sure your answer to 3 makes sense.

5. Compute the following integrals.
   (a) $\displaystyle\int \frac{dt}{\sqrt{t^2-6t+13}}$
   (b) $\displaystyle\int_{\pi/2}^{\pi} x\cos x\sin x\,dx$
   (c) $\displaystyle\int \tan^5\theta\sec^3\theta\,d\theta$
   (d) $\displaystyle\int \frac{2x^2+x+2}{x^3+2x-3}\,dx$
   (e) $\displaystyle\int (1+\ln x)\sqrt{1+(x\ln x)^2}\,dx$
   Wolfram Alpha won't do the above integral! But you can! Humans: 1 Computers: 11billion
   (f) $\displaystyle\int \frac{3x^2+2x+4}{x^3+x^2+4x-17}\,dx$
   (g) $\displaystyle\int \sin^4 x\,dx$
   (h) $\displaystyle\int \frac{dx}{\sqrt{1+\sqrt[3]{x}}}$
   (i) $\displaystyle\int \frac{\sec^2\theta\tan^2\theta}{\sqrt{9-\tan^2\theta}}\,d\theta$

(j) $\displaystyle\int \cos\theta\sqrt{1+\sin^{1/2}\theta}\,d\theta$

(k) $\displaystyle\int \arcsin x\,dx$

(l) $\displaystyle\int e^x\cos x\,dx$

(m) $\displaystyle\int \frac{dx}{x^3-12x^2+24x-8}$

(n) $\displaystyle\int \frac{dx}{x^4+2x^2+1}$

(o) $\displaystyle\int \frac{dx}{e^x+e^{-x}}$

(p) $\displaystyle\int x^3\sin x\,dx$

(q) $\displaystyle\int \tan^4 x\,dx$