Workshop 5   September 8, 2011

1. Here's a fairly epic use of partial fraction decompositions. We'll evaluate

$$\int_2^5 \frac{8x^2}{(x-1)^2(x^2+1)^2}\, dx.$$

   (a) First, write down the form of the partial fraction decomposition. For the sake of consistency, use capital letters A-F, and check with me before proceeding.

   (b) Now multiply through by the common denominator. You should end up with a long-ish equation of polynomials. You *could* expand everything out, equate coefficients of each power of x, and solve the resulting system of equations, but DON'T DO IT. I'll show you some small tricks to cut the amount of work down.

   (c) Notice all those $x-1$ terms floating around? Plug in $x = 1$ and simplify the resulting equation. This should tell you what B is.

   (d) There are also a lot of $x^2 + 1$ terms around. Since $x^2 + 1$ is unfactorable over the reals, you can't plug in a real number to kill it. You can, however, plug in $i = \sqrt{-1}$. (This is the only time we'll ever want to use complex numbers; you don't have to, but it can be helpful.) This should tell you what both E and F are.

   (e) Now we're out of tricks involving plugging in numbers for x. We're left with comparing coefficients, but at least now we won't have to use all of them. There are only a handful of terms that give an x^5, so find them and get an equation involving A and C. (In general, the highest degree term is a good choice.)

   (f) The constant term is also usually pretty easy; use the constant terms to find an equation that relates A and D.

   (g) Now you get a choice. The x^4 term and the x term are comparable in difficulty to extract. Pick one to get another equation. This equation together with the previous results should be enough to solve for all remaining variables.

   (h) Whew! That was fun! (Right?) Now all we have to do is integrate! (This shouldn't take long; the integrals you end up with aren't particularly difficult.)

   (i) Here's another part! I just wanted to get to (i).

2. Look! A chicken! You've already finished number 1, right?

3. Here's a simple one for a break. Evaluate

$$\int_{-1}^1 \frac{dx}{x^4}.$$

4. It's a choose-your-own adventure! Is your answer to 3 negative or zero? If so, go to question 5. If not, go to 6 on the back of the page.

5. You're eaten by an alligator! Okay, maybe I lied about 3 being simple. Take a look at the graph of $1/x^4$ and tell me why your answer shouldn't be negative. Where did you go wrong? (Hint: if you have a book handy, look up and *carefully* read the Fundamental Theorem of Calculus.) Then proceed to 7 on the back of the page.

6. So you know about improper integrals already? If not, go back to 5. If so, go on to 7.

7. You really made it this far today? Compute

   (a) $\displaystyle\int \frac{dx}{\sqrt{1+x}+\sqrt{x}}$

   (b) $\displaystyle\int \frac{1+2x+\arctan x}{x^2+1}\,dx$