Workshop 4 September 6, 2011

1. *A slightly different approach to trig substitutions*

   (a) Draw a right triangle with angle θ such that $\sec\theta = \frac{2x}{3} = \frac{x}{3/2}$.

   (b) Give another name for the quantity $\dfrac{\sqrt{x^2-9/4}}{x}$.

   (c) Can you use this information to evaluate $\displaystyle\int \frac{\sqrt{x^2-9/4}}{x}\,dx$? How about $\displaystyle\int \frac{3/2}{\sqrt{x^2-9/4}}\,dx$? (If necessary, use arcsin, arccos, or arctan, but avoid using arcsec, arccsc, and arccot.)

2. Here's an integral that can be completed in several different ways. Try different techniques and see where they take you.

$$\int xe^{\sqrt{1-x^2}}\,dx$$

3. *Fractions!*

   You're about to learn a theorem whose proof is well beyond the scope of this course. Let's at least see it in action.

   In the last two workshops, you worked on combining three fractions into one, then reversed that simplification to help with an integral that otherwise would have seemed insurmountable. Most rational functions behave like this: we want to be able to break them into sums of simpler rational functions.

   (a) If you're given $\dfrac{1}{(x+1)(2x+3)}$, what might the smaller fractions look like (think about how you would combine two fractions to get a denominator like this one)? Using that guess, try to figure out appropriate numerators. Make sure you check that your answer is correct!

   (b) What if you're given $\dfrac{1}{(x+2)(x^2+1)}$? Notice that the quadratic expression here doesn't factor, so what might the smaller fractions look like? Be careful about what the numerators might be like (in particular, think about whether you can break down $x/(x^2+1)$ into smaller fractions).

   (c) You'll need to be comfortable with long division of polynomials (or another technique that accomplishes the same thing). Rewrite

$$\frac{4x^5-2x^4-x^3+4x-6}{x^4-2x^2+x-1}$$

   as a polynomial plus a "proper" rational function. ("Proper" here is meant analogously to what it meant for fractions in grade school: a rational function $f(x)/g(x)$ is *proper* if the degree of f is strictly less than the degree of g.)

   (d) Integrate the functions given in parts (a) and (b) by using your decomposition.

   (e) Introduce some appropriate symbols to decompose

$$\frac{3x^3+8x+4}{(x^2+2x+3)(x-2)(x+2)}$$

   into simpler fractions. Then integrate.