Workshop 3 September 1, 2011

1. *Wrapping up from last time*

   Using problem 1 from workshop 2, evaluate

   $$\int \frac{3x^4 + 19x^3 + 44x^2 + 43x + 16}{(2x+3)(x^2+2x+2)(x^2+4x+5)}\, dx.$$

   (You might save yourself some time by also looking at workshop 1.)

2. *Some finer details about trig substitutions*

   (a) Compute $\int \sqrt{9 - x^2}\, dx$. You should use a trig substitution (say with angles t). Describe the domains for both variables (x and t); do you have a choice for the domain of t? Which choice(s) is(are) most convenient?

   (b) Use your indefinite integral (in terms of x) to compute $\int_{-3}^{3/2} \sqrt{9 - x^2}\, dx$.

   (c) Now compute the same definite integral, but use t-values as limits of integration. Note that in this method, you avoid computing sides of a triangle.

   (d) What if you used the substitution $x = 3\cos t$? What happens to the allowed t values? What t-bounds can you use to compute $\int_{-3}^{3/2} \sqrt{9 - x^2}\, dx$ with this substitution?

   (e) Recall that $\sqrt{u^2} = |u|$, not just u (why?). Why can we ignore this in the last two parts?

3. *A slightly different approach to trig substitutions*

   Draw a right triangle with angle θ such that $\sec\theta = \frac{2x}{3} = \frac{x}{3/2}$.

   (a) Give another name for the quantity $\dfrac{\sqrt{x^2 - 9/4}}{x}$.

   (b) Can you use this information to evaluate $\int \dfrac{\sqrt{x^2 - 9/4}}{x}\, dx$? How about $\int \dfrac{3/2}{\sqrt{x^2 - 9/4}}\, dx$? (If necessary, use arcsin, arccos, or arctan, but avoid using arcsec, arccsc, and arccot.)

4. *A look ahead*

   You're about to learn a theorem whose proof is well beyond the scope of this course. Let's at least see it in action.

   Last time, you worked on combining three fractions into one. This time, we've seen that integrating the three individual parts can be done (perhaps with some work, but we'll get used to that); the large fraction seems pretty insurmountable without all that algebraic work. This happens quite a bit, so we need some way to break big fractions down into smaller chunks.

   (a) If you're given $\dfrac{1}{(x+1)(2x+3)}$, what might the smaller fractions look like (think about how you would combine two fractions to get a denominator like this one)? Using that guess, try to figure out appropriate numerators. Make sure you check that your answer is correct!

(b) What if you're given $\frac{1}{(x+2)(x^2+1)}$? Notice that the quadratic expression here doesn't factor, so what might the smaller fractions look like? Be careful about what the numerators might be like (in particular, think about whether you can break down $x/(x^2+1)$ into smaller fractions).

(c) You'll need to be comfortable with long division of polynomials (or another technique that accomplishes the same thing). Rewrite

$$\frac{4x^5 - 2x^4 - x^3 + 4x - 6}{x^4 - 2x^2 + x - 1}$$

as a polynomial plus a "proper" rational function. ("Proper" here is meant analogously to what it meant for fractions in grade school: a rational function $f(x)/g(x)$ is *proper* if the degree of f is strictly less than the degree of g.)