Name: ____________________

- You have fifty minutes to complete this mock exam.

1. Find the sum of the series

$$\frac{1}{4}\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n+1}}{16^n(2n+1)!}.$$

   A. 0
   B. $\frac{1}{2}$
   C. $\frac{1}{\sqrt{2}}$
   D. $\frac{\sqrt{3}}{2}$
   E. The series does not converge.

2. Find the sum of the series

$$\sum_{n=0}^{\infty}\binom{\frac{1}{2}}{n}2^n.$$

   A. $\sqrt{3}$
   B. $\sqrt{2}$
   C. $\frac{1}{\sqrt{3}}$
   D. $\frac{1}{\sqrt{2}}$
   E. The series does not converge.

3. Which of the following is the Maclaurin series of $x\arctan\left(3x^2\right)$?

A. $\sum_{n=0}^{\infty}(-1)^n\frac{3^{2n+1}x^{4n+2}}{2n+1}$

B. $\sum_{n=0}^{\infty}(-1)^n\frac{3^{2n+1}x^{4n+3}}{2n+1}$

C. $\sum_{n=0}^{\infty}(-1)^n\frac{3^{4n+3}x^{4n+3}}{2n+1}$

D. $\sum_{n=0}^{\infty}(-1)^{2n+1}\frac{3^{2n+1}x^{4n+3}}{2n+1}$

E. $\sum_{n=0}^{\infty}(-1)^n\frac{3^{2n+1}x^{2n+1}}{2n+1}$

4. Evaluate the limit

$$\lim_{x\to 0}\frac{\sin x - x + \frac{1}{6}x^3}{x^5}.$$

A. 0

B. $-\frac{1}{3!}$

C. $\frac{1}{5!}$

D. $-\frac{1}{7!}$

E. $\frac{1}{9!}$

5. If the Maclaurin series for the function $f(x) = \sqrt{1+x}$ is used to approximate $\sqrt{1.1}$, what is the minimum number of terms required to achieve three decimal places of accuracy?

   A. One

   B. Two

   C. Three

   D. Four

   E. Five

6. Do the following series converge or diverge?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n}{3^n + 5^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$

7. Find the interval of convergence for each of the following series.

(a) $\sum_{n=1}^{\infty} n^n x^n$

(b) $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$

8. Consider the function

$$f(x) = \frac{e^{x^2}}{1 - x^2}.$$

(a) Write the first three nonzero terms in the Maclaurin series of $f$.

(b) Compute $f^{(3)}(0)$ and $f^{(4)}(0)$.

9. Compute
$$\sum_{n=1}^{\infty} \frac{n}{2^n}.$$

10. Let $f$ be a function that is infinitely differentiable at $x = a$.

(a) Define the Taylor series of $f$ centered at $x = a$.

(b) Define the degree-$N$ Taylor polynomial of $f$ centered at $x = a$.

(c) Define the Taylor remainder $R_N(x)$.

(d) What does Taylor's theorem say about $R_N(x)$?