Name: ______________________________

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have until 4:30pm to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.
- Make sure to check whether you are asked to *set up* or to *evaluate* integrals.

Some possibly useful formulas:

$$\cos^2 t = \frac{1}{2}(1 + \cos(2t))$$
$$\sin^2 t = \frac{1}{2}(1 - \cos(2t))$$
$$\sin(2t) = 2\sin(t)\cos(t)$$

1. Consider the planes $2x - y + z = 4$ and $x + 3y - 2z = 9$.
   (a) Are these planes parallel, perpendicular, or neither? How do you know?

   (b) If they are parallel, find the distance between them. Otherwise find the equation of the line that is their intersection.

2. Consider the planes $2x - y + z = 4$ and $-4x + 2y - 2z = 7$.
   (a) Are these planes parallel, perpendicular, or neither? How do you know?

   (b) If they are parallel, find the distance between them. Otherwise find the equation of the line that is their intersection.

3. A particle moves in space, with position at time t given by (t, t^2, t^3).
   (a) Find the velocity at time $t = 1$.

   (b) Find the acceleration at time $t = 1$.

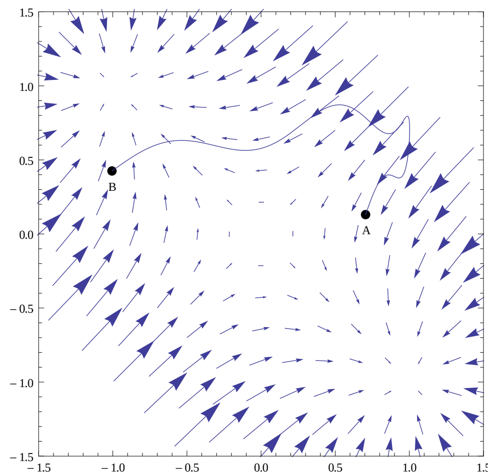
   (c) Find the tangential and normal components of acceleration at time $t = 1$. Is the particle speeding up or slowing down at time $t = 1$?

4. Let R be the region in the first quadrant bounded by the curves $xy = 1$, $xy = 2$, $y = x$, and $y = 3x$. Transform $\iint_R x^2 \, dx \, dy$ into an integral over a rectangle, and evaluate.

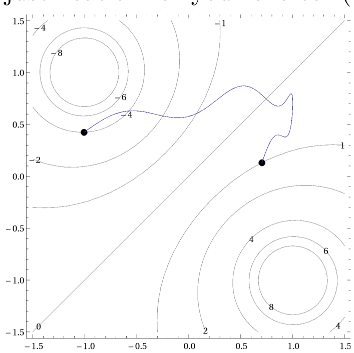
5. Compute the flow of $\mathbf{F}(x, y, z) = \langle yz^2 + e^z + x, ze^z + x + y, xe^x + xy + z \rangle$ across the sphere S given by $x^2 + y^2 + z^2 = 1$. Which direction is it?

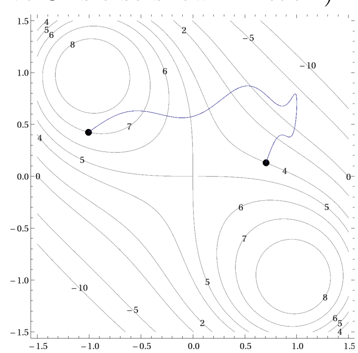
6. Compute the volume of the "polar cap" bounded above by the hemisphere $z = \sqrt{25 - x^2 - y^2}$ and below by the plane $z = 4$. (Hint: this is not a "rectangle" in any of our coordinate systems: some variables have to depend on others.)

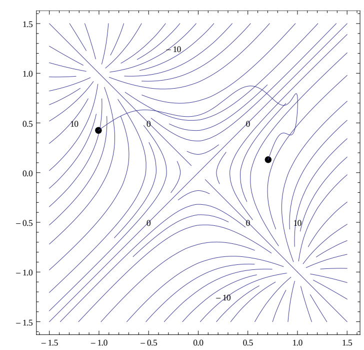
7. To the right is shown part of a gradient (a.k.a. conservative) field **F**, together with a curve C.

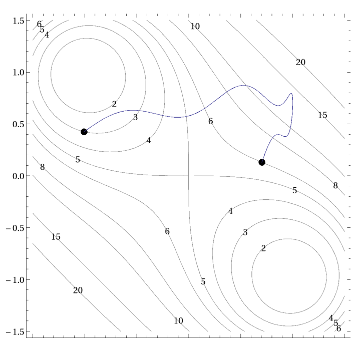


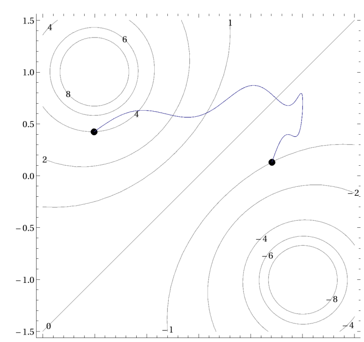
(a) One of the following is the contour plot of a potential function for **F**. Circle it. Give a brief justification for your choice. (The curve C is also shown in each.)







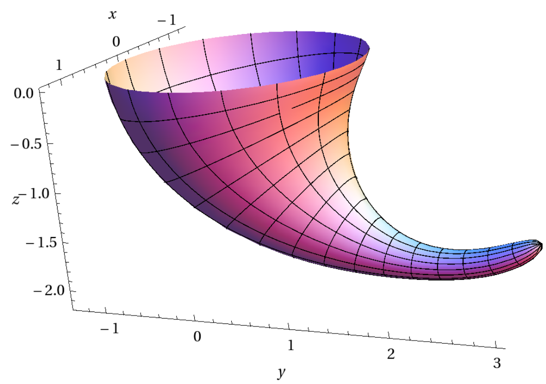


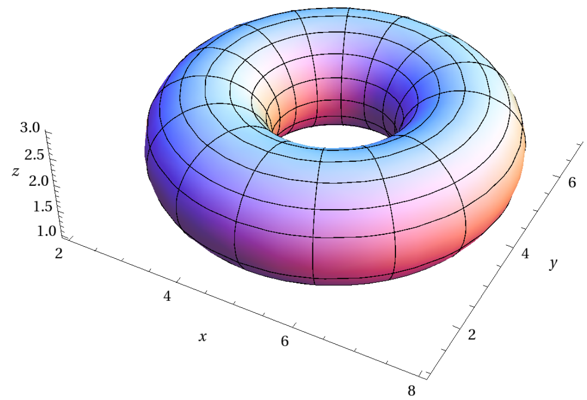


(b) Find and classify all critical points of the potential function in the region shown.

(c) Find $\int_C \mathbf{F} \cdot \langle dx, dy \rangle$, assuming C is parametrized to go from A to B.

8. Below is shown a surface R whose boundary is the unit circle in the xy-plane, and a surface S that has no boundary. Let $\mathbf{G}(x, y, z) = \langle x, y, z \rangle$. For each of the following, explain your answer.





(a) Which direction is the net flow of $\mathbf{G}$ across S?

(b) Which direction is the net flow of $\mathbf{G}$ across R?

(c) Which direction is the net flow of $\text{curl}(\mathbf{G})$ across R?