UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
MATH 241 C8 AND X8 - FINAL EXAM

Name: ________________________

NetID: ________________________

Section: (circle one) C8 X8

May 8, 2013

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Read all of the following information before starting the exam:

- **Do NOT open the exam until instructed to do so.**
- You have until 4:30pm to complete this exam. When you are told to stop writing, do so or you will lose all points on the page you write on.
- You may not communicate with other students during this exam.
- Show all work, clearly and in order, if you want to get full credit. Keep written answers clear, concise, and to the point. The graders reserve the right to take off points if they cannot see how you arrived at your answer (even if your final answer is correct), or if your answer makes no sense as written.
- **Written materials, calculators, phones, and other aids including all electronic devices, are NOT permitted for this test.** No scratch paper is allowed except that given to you by the proctors. While a wristwatch is fine, the time will also be made available for you by the proctors.
- Before turning in your exam, check to make certain that you have answered all of the questions.
- Make sure to check whether you are asked to *set up* or *calculate* integrals.
- This test has 10 problems, 17 pages, and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

Problem	**Grade**
1. (/10)	
2. (/10)	
3. (/10)	
4. (/10)	
5. (/10)	
6. (/10)	
7. (/10)	
8. (/10)	
9. (/10)	
10. (/10)	
Total: (/100)	

Problem 1 (Quick Calculations – 10 points).

(a) Let $f(x, y, z) = \cos^2(xy) + z^3 y e^{xz}$. Find $\frac{\partial^2 f}{\partial z \partial x}$.

(b) Parametrise the ellipse given by the equation $\frac{(x-2)^2}{3^2} + \frac{y^2}{2^2} = 1$.

(c) Find the volume of the parallelepiped determined by the three vectors $\langle 2, 3, 5\rangle$, $\langle -1, 0, 4\rangle$, and $\langle -2, -2, -1\rangle$.

Problem 2 (Planes – 10 points)**.** Consider the planes $4x+3y-2z=6$ and $-3x+y-3z=2$.

(a) Are these planes parallel, perpendicular, or neither? How do you know?

(b) If they are parallel, find the distance between them. Otherwise, find the equation of the line that is their intersection.

Problem 3 (Velocity and Acceleration – 10 points)**.** Let the position of a particle at time t be given by $\langle \cos(t), \sin(t), t^2 \rangle$.

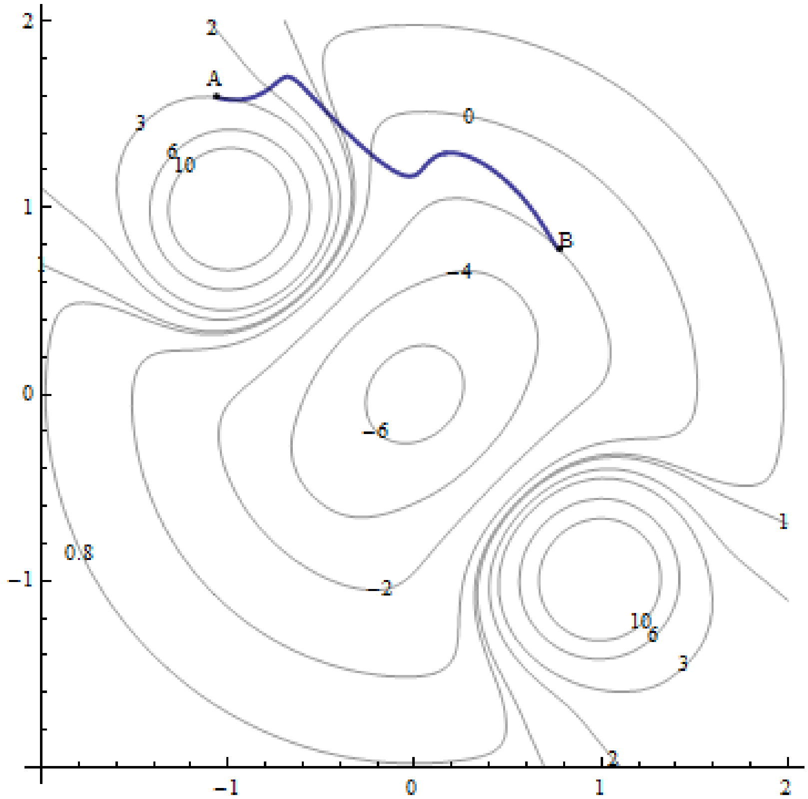
(a) Find the velocity at time $t = \pi$.

(b) Find the acceleration at time $t = \pi$.

(c) Find the tangential and normal components of the acceleration at time $t = \pi$. Is the particle speeding up or slowing down at time $t = \pi$?

Problem 4 (Optimisation – 10 points)**.** Find the absolute maximum and minimum of $f(x, y) = x^2 + y^2 + y$ on the disc $x^2 + y^2 \leq 1$. (Remember to check both the inside of the disc and its boundary.)

Problem 5 (Contour Map – 10 points)**.** Below is shown a contour map of a function $f(x, y)$ on the rectangle D. Let $\mathbf{F}(x, y) = \nabla f(x, y)$. Briefly explain all your answers.



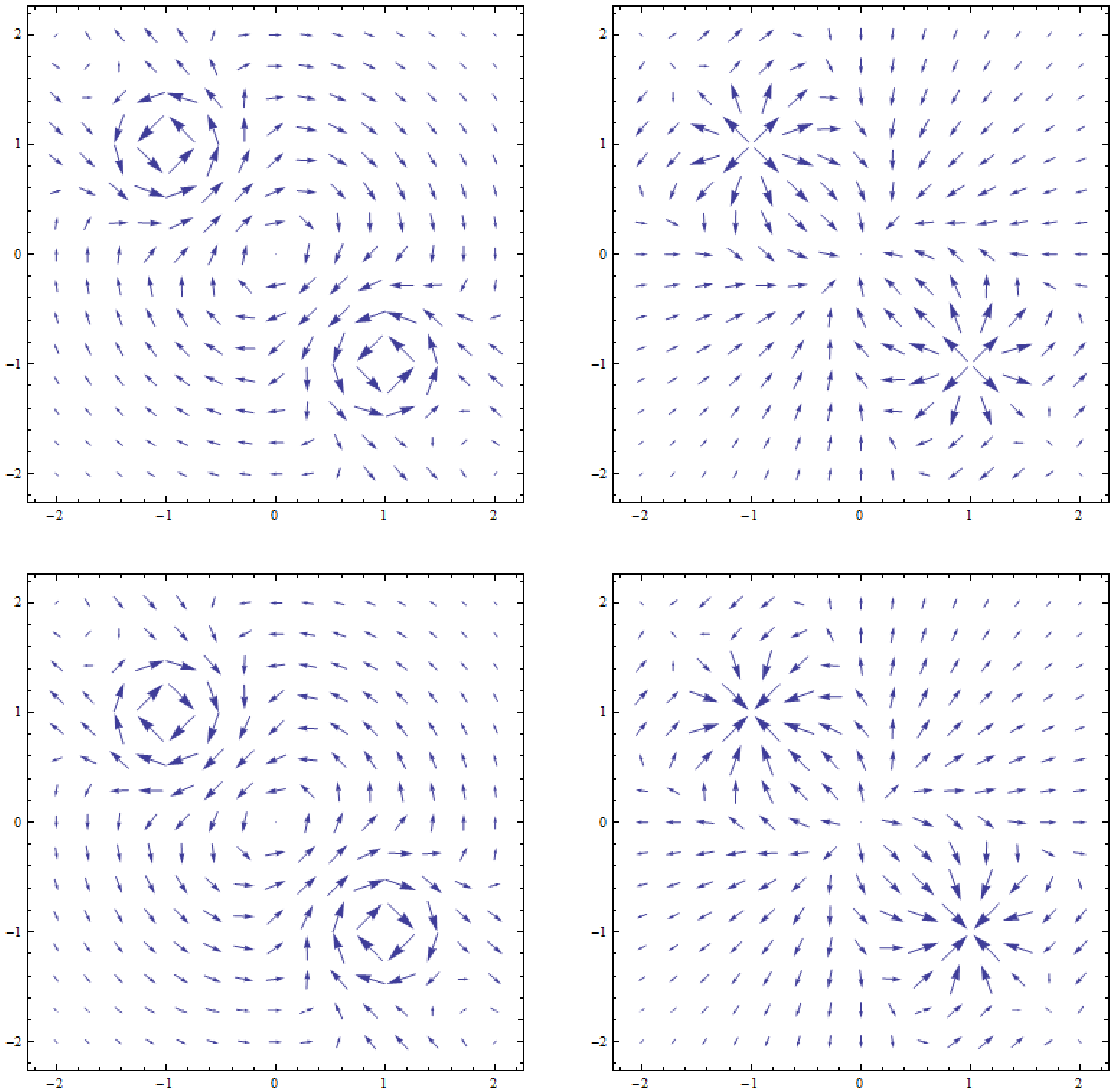
(a) Which of the following is the best estimate of $\iint_D f\, dx\, dy$:

$-40 \qquad -20 \qquad 0 \qquad 20 \qquad 40$

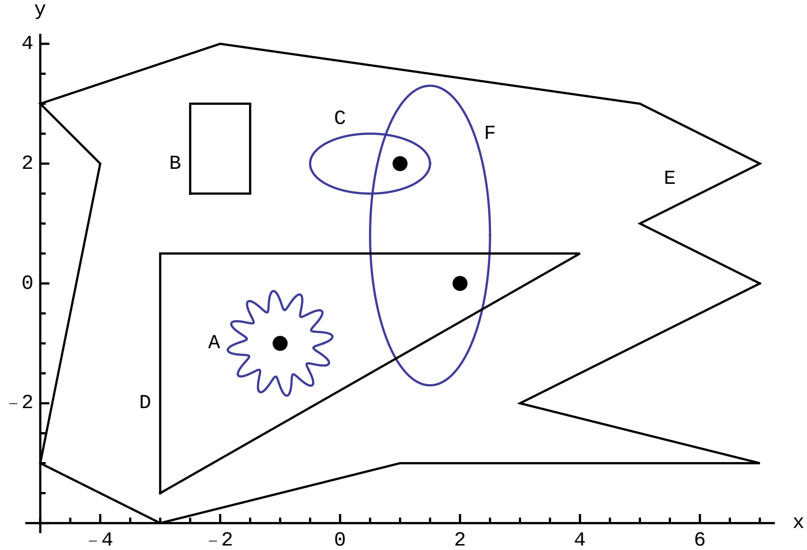
(b) Find $\int_C \mathbf{F} \cdot \langle dx, dy \rangle$, where C is the path going from A to B, as shown.

(c) Find and classify the critical points of f.

(d) Which of the following is $\mathbf{F}$?



Problem 6 (2D Flows of Vector Fields – 10 points)**.** A vector field $\mathbf{G}$ has zero divergence at every point except the three singularities $\langle -1,-1\rangle$, $\langle 1,2\rangle$, and $\langle 2,0\rangle$. Below are shown these singularities together with several curves. All curves are assumed to be parametrized counterclockwise.



You are given the following data:

$$\int_D \mathbf{G}\cdot\langle dy,-dx\rangle = -1 \qquad \int_E \mathbf{G}\cdot\langle dy,-dx\rangle = 3 \qquad \int_F \mathbf{G}\cdot\langle dy,-dx\rangle = 9$$

Use this information to compute each of the following. Give brief explanations.

(a) $\int_A \mathbf{G}\cdot\langle dy,-dx\rangle$

(b) $\int_B \mathbf{G}\cdot\langle dy,-dx\rangle$

(c) $\int_C \mathbf{G}\cdot\langle dy,-dx\rangle$

(d) Say as much as you can about which points in the plane are sources or sinks in $\mathbf{G}$.

Problem 7 (3D Change of Coordinates – 10 points)**.** Re-express the integral over the cone

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{2-\sqrt{x^2+y^2}} f(x,y,z)\,dz\,dy\,dx$$

using cylindrical coordinates.

Bonus (10 points): Express the integral in spherical coordinates.

Problem 8 (Surface Area – 10 points)**.** Set up a double integral that computes the surface area of the paraboloid $z = x^2 + y^2$ above the xy-plane, and below the plane $z = 4$. (Do not solve the integral.)

Problem 9 (Net Flow Across I – 10 points)**.** Let $\mathbf{F}(x, y, z) = \langle \frac{x}{x^2+y^2+z^2}, \frac{y}{x^2+y^2+z^2}, \frac{z}{x^2+y^2+z^2} \rangle$.

(a) Compute the divergence of $\mathbf{F}$.

(b) Compute the triple integral of the **divergence** of $\mathbf{F}$ over the solid region bounded between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$.

(c) Use the divergence theorem to express the net flow of $\mathbf{F}$ across the sphere $x^2+y^2+z^2 = 4$ in terms of the net flow across the sphere $x^2 + y^2 + z^2 = 1$.

Problem 10 (Net Flow Across II – 10 points)**.** Let $\mathbf{F}(x, y, z)$ be the vector field

$$\langle xz - y^3\cos(z), x^3e^z, ze^{x^2+y^2+z^2}\rangle.$$

Find the net flow of the **curl** of $\mathbf{F}(x, y, z)$ across the upper hemisphere of $x^2 + y^2 + z^2 = 1$. (Use Stokes' Theorem to replace the surface with an easier surface.)

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