Name: Solutions

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have until 10:50am to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.

Question:	1	2	3	4	5	Total
Points:	20	25	15	20	20	100
Score:						

1. Consider the planes $3x - y + 2z = 4$ and $2x - 2y + z = 6$.
   (a) (5 points) Explain why, at a glance, you know these planes are not parallel.

   Their normal vectors, $\langle 3,-1,2\rangle$ and $\langle 2,-2,1\rangle$ are not parallel.

   (b) (8 points) Find a vector that is parallel to both planes.

   The cross product of their normal vectors will work:

   $$\langle 3,-1,2\rangle \times \langle 2,-2,1\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix}$$

   $$= (-1+4)\vec{i} - (3-4)\vec{j} + (-6+2)\vec{k}$$

   $$= \boxed{\langle 3, 1, -4\rangle}$$

   (c) (7 points) Give an equation for the line that is the intersection of the two planes.

   (b) gives us a direction vector, so we just need a point.

   Arbitrarily try $x=0$: $\begin{cases} -y+2z=4 & ① \\ -2y+z=6 & ② \end{cases}$

   $2\cdot① - ②$: $3z = 2$

   $z = 2/3 \xrightarrow{①} y = 2z - 4 = \frac{4}{3} - 4 = -\frac{8}{3}$

   So $\boxed{\ell(t) = \left(0, -\frac{8}{3}, \frac{2}{3}\right) + t(3, 1, -4)}$.

2. Suppose a particle moves in the plane, with position $(t^2, 2t^3)$ at time t.

(a) (3 points) Find the velocity at time $t = 1$.

$\langle 2t, 6t^2 \rangle$ at $t=1$: $\langle 2, 6 \rangle$

(b) (3 points) Find the acceleration at time $t = 1$.

$\langle 2, 12t \rangle$ at $t=1$: $\langle 2, 12 \rangle$

(c) (7 points) Find the tangential component of acceleration at time $t = 1$.

$$\frac{\langle 2,6\rangle \cdot \langle 2,12\rangle}{\langle 2,6\rangle \cdot \langle 2,6\rangle}\langle 2,6\rangle = \frac{4+72}{4+36}\langle 2,6\rangle$$

$$= \frac{76}{40}\langle 2,6\rangle$$

$$= \frac{19}{10}\langle 2,6\rangle$$

$$= \langle 3.8, 11.4\rangle$$

(d) (7 points) Find the normal component of acceleration at time $t = 1$.

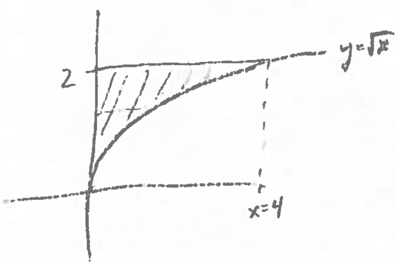
$\langle 2, 12\rangle - \langle 3.8, 11.4\rangle = \langle -1.8, 0.6\rangle$

(e) (5 points) What do the above tell you about how the speed of the particle is changing at $t = 1$?

From (c), acceleration is generally in the same direction of velocity; hence the speed is increasing at $t=1$.

3. (15 points) By switching the order of integration, compute

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{3}{4+y^3}\,dy\,dx.$$



$$= \int_0^2 \int_0^{y^2} \frac{3}{4+y^3}\,dx\,dy$$

$$= \int_0^2 \frac{3y^2}{4+y^3}\,dy \qquad u = 4+y^3, \quad du = 3y^2\,dy$$

$$= \int_4^{12} \frac{du}{u}$$

$$\boxed{= \ln 12 - \ln 4}$$

$$= \ln 3$$

4. (20 points) Find the maximum and minimum values of $f(x, y) = x^2 + 9y^2$ on the disk $x^2 + y^2 \leq 4$. Hint: consider the interior of the disk and its boundary (the circle) separately. Then sketch the region together with the level curves for f corresponding to your maximum and minimum.

Interior: $\nabla f = \langle 2x, 18y \rangle = \langle 0, 0 \rangle \iff \langle x, y \rangle = \langle 0, 0 \rangle$

$$\boxed{f(0,0) = 0 \quad \text{Min}}$$

Boundary: Lagrange:

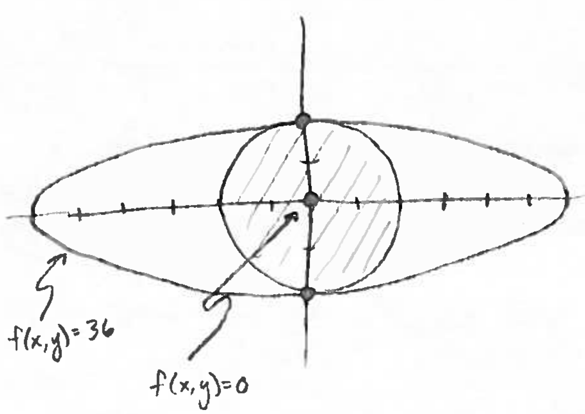
$$\begin{cases} 2x = \lambda(2x) \Rightarrow \lambda = 1 \text{ or } x = 0 \\ 18y = \lambda(2y) \\ x^2 + y^2 = 4 \end{cases}$$

$\lambda = 1 \Rightarrow y = 0 \Rightarrow x = \pm 2$

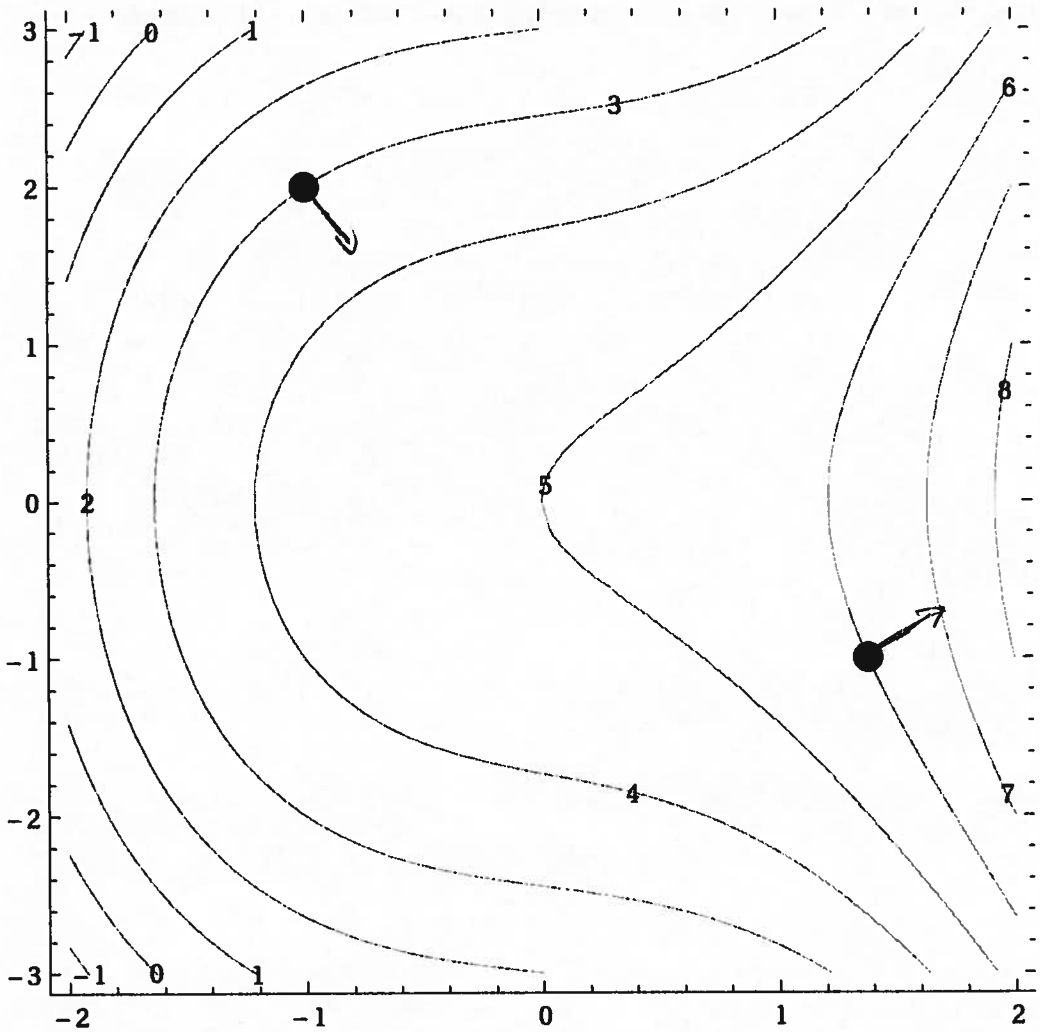
$x = 0 \Rightarrow y = \pm 2$

$f(\pm 2, 0) = 4$

$$\boxed{f(0, \pm 2) = 36 \quad \text{Max}}$$



5. Below is a plot of several level curves of a function $f(x,y)$ inside the rectangle R.



(a) (15 points) At each of the two indicated points, sketch in the gradient vectors.

(b) (5 points) Which of the following intervals does $\iint_R f(x,y)\,dx\,dy$ fall into? Explain how you know.

$(-\infty,-120)$ $(-120,-80)$ $(-80,-40)$ $(-40,0)$ $(0,40)$ $(40,80)$ $(80,120)$ $(120,\infty)$

f is overwhelmingly positive on R, so $\iint_R f\,dx\,dy > 0$.

To approximate the value, notice that the average value of f on R seems to be between 4 and 5. Thus

$\iint_R f\,dx\,dy = \text{Area}(R)\cdot$ "average height", $\underset{96}{24\cdot 4} \lesssim \iint_R f\,dx\,dy \lesssim \underset{120}{24\cdot 5}$