Name: ______________________________

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have until 10:50am to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.

Some possibly useful formulas:

$$\cos^2 t = \frac{1}{2}(1 + \cos(2t))$$
$$\sin^2 t = \frac{1}{2}(1 - \cos(2t))$$

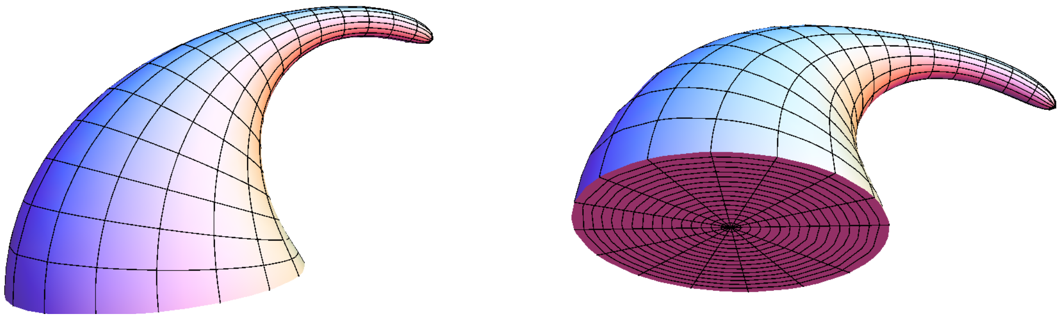
1. Let $\mathbf{F}(x, y) = \langle ye^x + x^2 + yz^2, e^x + z + xz^2, y + 2xyz + \cos(z)\rangle$.
   (a) Verify that $\mathbf{F}$ is a gradient field.

   (b) Find a potential function for $\mathbf{F}$.

   (c) Compute the flow of $\mathbf{F}$ along the part of the curve $x = \sin(\pi t)$, $y = e^t$, $z = t^3$ going from $(0, 1, 0)$ to $(0, e, 1)$.

2. Let $\mathbf{F}(x, y, z) = \langle 3xz^2, y^3, 3x^2z \rangle$. The surface R consists of two parts: the part of the cone $z = \sqrt{x^2 + y^2}$ with $z \leq 1$ and the part of the sphere $x^2 + y^2 = 1$ inside the cone. Compute the flux of $\mathbf{F}$ across R.

3. Let $\mathbf{F}(x, y, z) = \langle xze^{xyz}, -yze^{xyz}, -2\rangle$. The surface R consists of two parts: the disk R_1 given by $x^2 + y^2 \leq 1$ in the xy-plane, and the "hat" surface R_2. This is pictured below from two different perspectives.



   (a) Explain why the flow of $\mathbf{F}$ across R is zero.

   (b) Compute the flow of $\mathbf{F}$ across R_1. (Which direction is it?)

   (c) Find the flow of $\mathbf{F}$ across R_2. (Which direction is it?)

4. Let $\mathbf{G} = \langle y, -x, e^{xyz} \rangle$.
   (a) Compute $\operatorname{curl} \mathbf{G}$.

   (b) Using (a), give another quick solution to question 3(b). (Be careful about whether the sign is correct!)

5. A very linear-minded student gets stumped by the following integral. Think nonlinear and compute the integral for them.

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} \sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$$

6. Find the volume of the region bounded by the planes

$$\begin{array}{lll} x+y+z=1 & x+2y=-1 & y+z=2 \\ x+y+z=4 & x+2y=1 & y+z=4 \end{array}$$

7. Use Stokes's Theorem to compute the flow of $\mathbf{F}(x, y, z) = \langle xy, 2z, 3y \rangle$ along C, where C is the curve of intersection of the cylinder $x^2 + y^2 = 9$ and the plane $x + z = 5$. When viewed from above (from large positive z), is the flow clockwise or counterclockwise?

8. Compute $\iiint_E z\,dV$, where E is enclosed by the surface $z = x^2 + y^2$ and the plane $z = 4$.