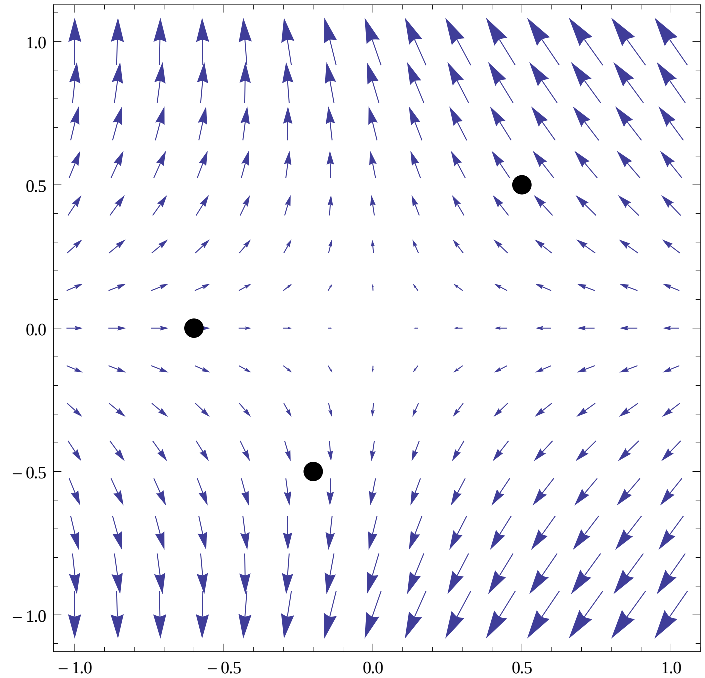
Name: ______________________________

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have until 10:50am to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.

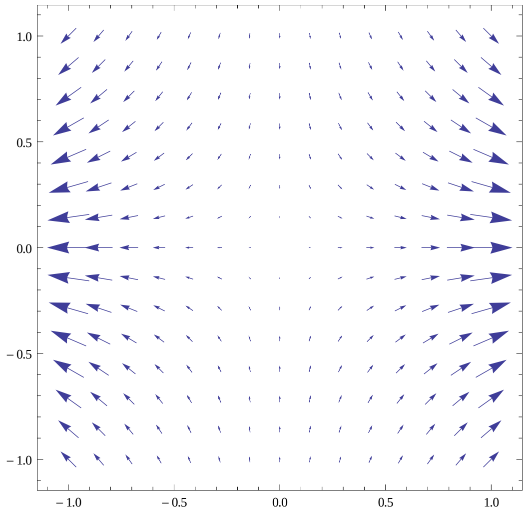
Some possibly useful formulas:

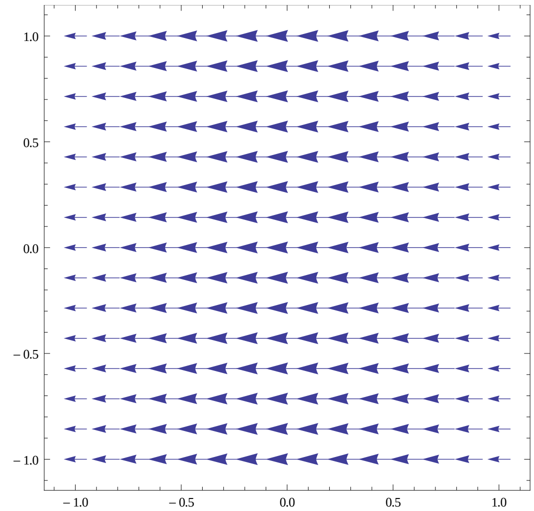
$$\cos^2 t = \frac{1}{2}(1+\cos(2t))$$
$$\sin^2 t = \frac{1}{2}(1-\cos(2t))$$

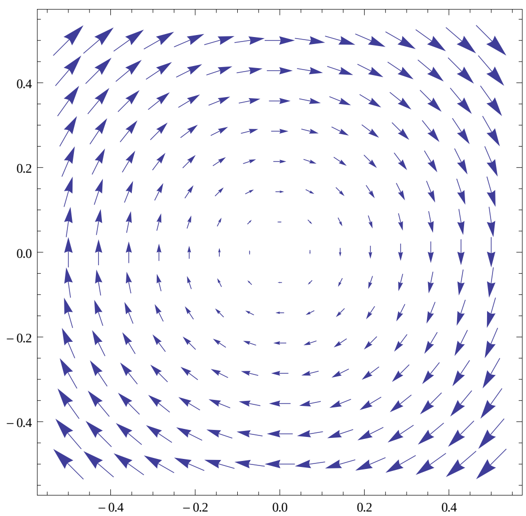
1. Sketch the trajectories that pass through the indicated points below.

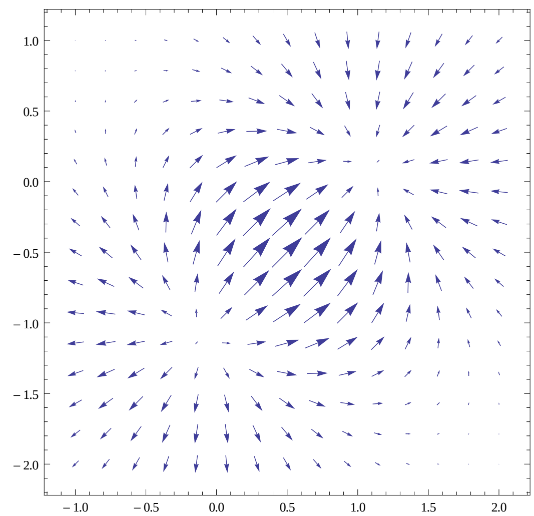


2. All but one of the following vector fields are gradient fields. Which one can't be a gradient field? **Why?**









3. Let $\mathbf{F}(x, y) = \langle ye^x + \frac{1}{x^2}, e^x + y^4 \rangle$.

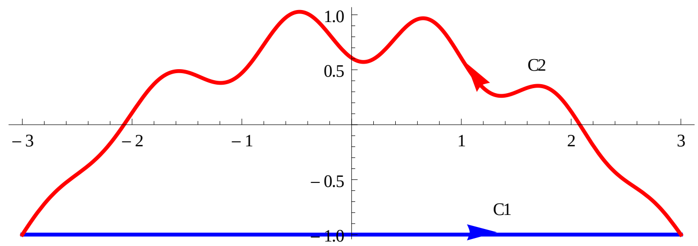
   (a) Find a potential function for $\mathbf{F}$.

   (b) Compute the flow of $\mathbf{F}$ along the part of the curve $y = \sin(\pi x)$ going from $(0, 0)$ to $(3, -1)$.

4. Find all sources and sinks of the vector field $\mathbf{F}(x, y) = \langle y^4, xy^3e^x \rangle$.

5. Find all sources and sinks of the vector field $\mathbf{G}(x, y) = \left\langle \frac{x+y}{x^2+y^2}, \frac{y-x}{x^2+y^2} \right\rangle$.

6. Let $\mathbf{F}(x, y) = \langle xy^2, x^2 y \rangle$. The curve C consists of two parts, C_1 and C_2, as shown below.



(a) Find the flow of $\mathbf{F}$ along C.

(b) Find the flow of $\mathbf{F}$ along C_1.

(c) Find the flow of $\mathbf{F}$ along C_2.

7. Let $\mathbf{F}(x, y) = \langle 2, \frac{1}{2}y^2 \rangle$. Let C be the curve going from $(1, 0)$ to $(0, 2)$ along the parabola $4 - 4x = y^2$, then from $(0, 2)$ to $(-1, 0)$ along the parabola $4 + 4x = y^2$, then from $(-1, 0)$ to $(1, 0)$ along the x-axis. Let R be the region bounded by C.

   (a) Explain why you can measure the flow of $\mathbf{F}$ across C by the double integral

$$\iint_R y \, dx \, dy.$$

   (b) Use the transformation $x = u^2 - v^2$ and $y = 2uv$ to compute the above integral.

8. Let R be the region in the first quadrant bounded by the curves $x^2 - y^2 = 1$, $x^2 - y^2 = 4$, $x - y = 1$, and $x - y = 3$. Compute the area of R.

9. Compute $\iint_D \frac{1}{\sqrt{x^2+y^2}}\,dx\,dy$ where D is the unit disk. (Remark: note that this is an improper double integral, but your choice of transformation turns it into a proper (and easily computed) iterated integral.)