Name: ____________________

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have until 10:50am to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.

Some possibly useful formulas:

$$\iint_R (\partial_x n - \partial_y m)\,dx\,dy = \int_a^b (mx' + ny')\,dt$$

$$\cos^2 t = \frac{1}{2}(1+\cos(2t))$$

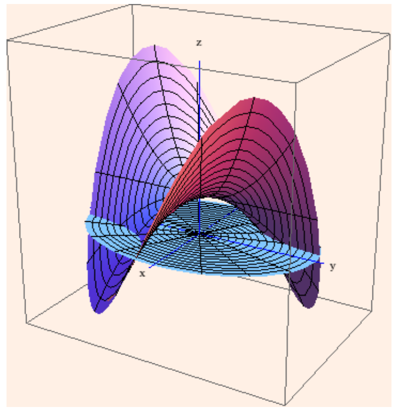
$$\sin^2 t = \frac{1}{2}(1-\cos(2t))$$

1. Consider the parametric equations $x = t^2$, $y = \sin(\pi t)$.

   (a) Find the slope(s) of the tangent line(s) to the curve at the point(s) where $x = 9$.

   (b) What are the minimum and maximum values taken by x and y in this curve?

2. Consider the surface plotted below. It is given by $z = f(x, y)$ over the region R inside the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, where $f(x, y) = xy + 1$.



(a) (8 points) Without doing any computations, do you think the integral $\iint_R f(x, y)\, dx\, dy$ is positive or negative? Why?

(b) (15 points) Compute $\iint_R f(x, y)\, dx\, dy$.

3. By switching the order of integration (careful about the bounds!), compute

$$\int_0^1 \int_x^1 e^{y^2}\, dy\, dx.$$

(Note that you don't know an antiderivative for e^{y^2} with respect to y, so you can't do this integral directly.)

4. Let $\vec{u} = \langle 2, 6 \rangle$ and $\vec{v} = \langle -2, -1 \rangle$. Compute and plot the following together with u and v.

   (a) $\vec{u} + \vec{v}$

   (b) $2\vec{v}$

   (c) the angle between $\vec{u}$ and $\vec{v}$

   (d) the push/projection of $\vec{u}$ in the direction of $\vec{v}$

5. Consider the lines $\ell_1(t) = (0, 1, 3) + t(2, 1, 5)$ and $\ell_2(t) = (-5, 2, 3) + t(-9, 9, 30)$. Are they parallel, perpendicular, or neither? Do they intersect or not?

6. Consider the lines $\ell_1(t) = (0, 1, 3) + t(2, 1, 5)$ from above and $\ell_3(t) = (3, 0, 1) + t(-2, -1, 1)$. Are they parallel, perpendicular, or neither? Do they intersect or not?

7. Consider the two planes given by equations

$$3x - y + z = 4$$
$$2x + y - 2z = 6.$$

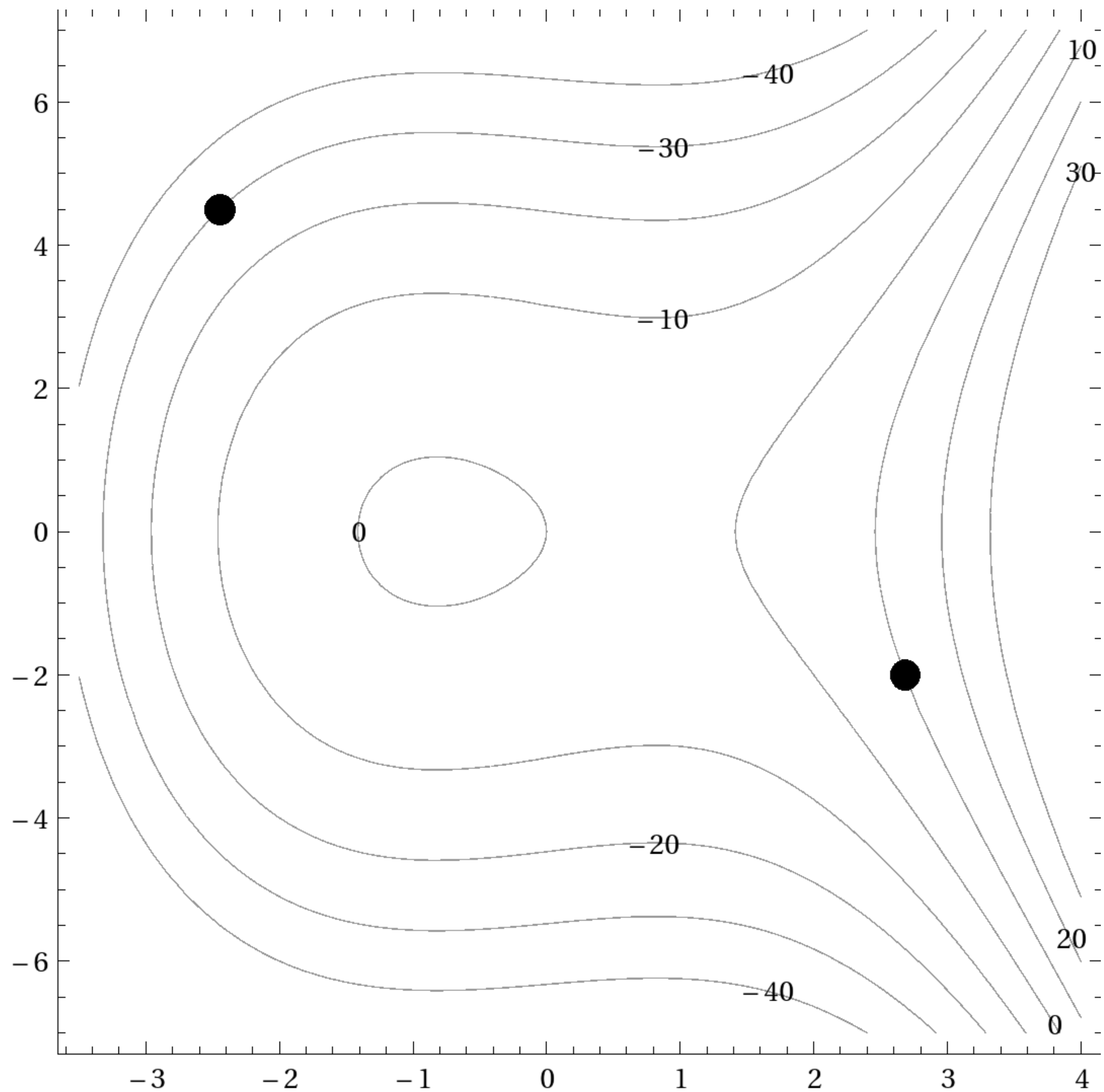
Find an equation of the line that is the intersection of these planes.

8. Give parametric equations for the unit circle in the plane $3x - y + z = 4$ centered at $(1, 0, 1)$.

9. Find the maximum and minimum values of $f(x, y) = xy$ on the disk $x^2 + y^2 \leq 4$. Hint: consider the interior of the disk and its boundary (the circle) separately.

10. Given the motion of a particle, why is acceleration(t) = speed(t) unittangent(t)? Use this and the chain rule to find the tangential and normal components of acceleration in terms of the unit tangent, unit normal, and unit binormal vectors.

11. Below is a plot of several level curves of a function $f(x, y)$. At the indicated points, sketch in the gradient vectors.



12. Consider the situation below, in which a ray of light is being sent from the focus $(1 + \frac{1}{\sqrt{2}}, 0)$ of the hyperbola $x^2 - y^2 = 1$ toward the point $(2, \sqrt{3})$ on the hyperbola. Provide a system of equations that could be solved to find the vector representing the path of the light after reflection off of the hyperbola. (Your system should have a unique solution.) Briefly explain the theory behind your answer. (Adding to the picture may be useful.)