Name: _____

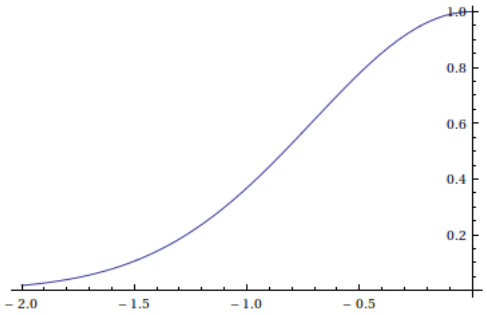
- **Read the following directions!**
- **Do NOT open the exam until instructed to do so.**
- You have until 4:30pm to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with anyone other than me during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.
- There are a few formulas on the last page of this exam.

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	12	12	12	12	12	12	12	16	12	12	16	23	163
Score:													

1. (12 points) In some situation, suppose a spline between two roads is considered safe it has order of contact at least 4. Below is shown a road following a curve whose expansion in powers of x is

$$1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \frac{x^8}{24} - \frac{x^{10}}{120} + \cdots .$$

Find a polynomial equation of a safe road that connects to the right end of the partial road shown (i.e., at the point $(0, 1)$. Roads are least expensive when they're not complicated, so choose the smallest degree polynomial possible.

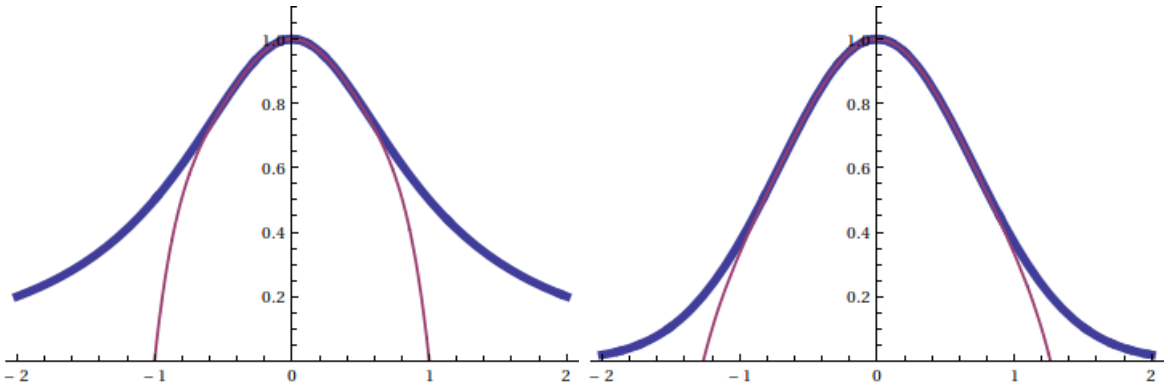


2. (12 points) Find the expansion of $\ln(1 + x)$ in powers of x; what is its radius of convergence?

3. (12 points) A certain mathematician finds that he proves theorems at a rate of $\frac{1}{(2t+1)^2}$ theorems per hour, where t is the amount of time since his last coffee. He starts his day at 8am with a coffee, gets more coffee at 11am and 3pm, and quits at 5pm. How many theorems does he prove during the day?

4. (12 points) Compute $\int_0^{\pi} e^x \cos(3x)\, dx$.

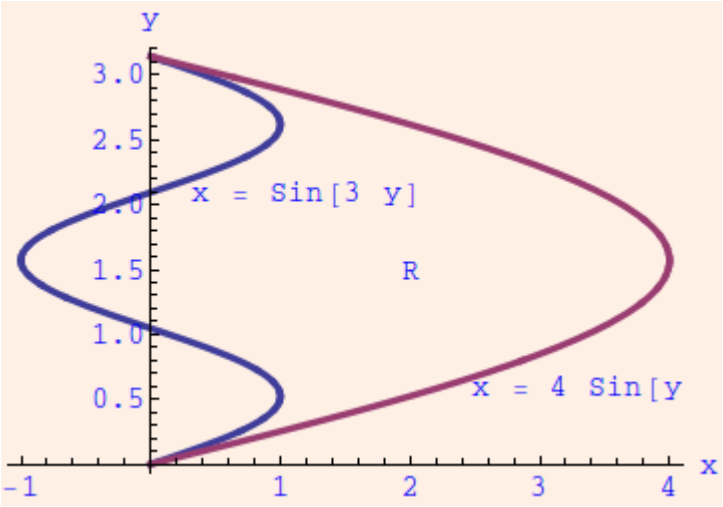
5. (12 points) Below are pictures of the curves $y = \dfrac{1}{1+x^2}$ (on the left, in bold) and $y = e^{-x^2}$ (on the right, in bold), together with their 6^{th}-degree partial expansions in powers of x (not in bold). One of the expansions can be improved near $x = 2$ by taking more terms, and the other cannot. Carefully and fully explain which one is which and why. (Hint: the pictures are here just for show; you needn't use them in your explanations.)



6. (12 points) Compute the following limit, making use of expansions.

$$\lim_{x \to 0} \frac{\sin(x) - x + \frac{1}{6}x^3}{x^5}$$

7. (12 points) Below is a picture of a region R; it is bounded by the curves $x = \sin(3y)$ and $x = 4\sin(y)$. Compute $\iint_R 1\,dx\,dy$, and say what it represents.



8. Consider the expansion of $f(x) = \sqrt{x}$ in powers of $x - 1$.

   (a) (8 points) What is the interval of convergence of this expansion?

   (b) (8 points) Find the first three terms of this expansion using Taylor's Theorem.

9. (12 points) You win the lottery, and the state offers you two payment options. You can get \$100 right now, or you can get \$12 per year every year forever, starting right now. Between inflation and lost interest payments, \$1 given to you t years from now is only really worth $\$(0.9)^t$. How much is this second payment option worth? Which one should you choose?

10. (12 points) Use the method(s) from class to determine whether $\frac{1}{3} + \frac{1}{8 \cdot 9} + \cdots + \frac{1}{k^3 3^k} + \cdots$ converges or not.

11. (a) (8 points) Compute the integral $\displaystyle\int_0^3 \frac{dx}{(x-4)(x+1)}$.

(b) (8 points) Explain why it is silly to try to compute $\displaystyle\int_0^3 \frac{dx}{(x-1)(x+4)}$.

12. Quickies:

(a) (6 points) What is the harmonic series? Does it converge?

(b) (4 points) What is the formula for the arc length of a parametric curve $(x(t), y(t))$, $a \leq t \leq b$?

(c) (5 points) Give formulas for the centroid of a region R.

(d) (8 points) Solve the differential equation $y' = y$, $y(0) = 1$.

Scratch Paper - you may remove this if you find it convenient

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Some possibly useful formulas:

$$\iint_R (\partial_x n - \partial_y m)\, dx\, dy = \int_a^b (mx' + ny')\, dt$$

$$\cos^2 t = \frac{1}{2}(1 + \cos(2t))$$

$$\sin^2 t = \frac{1}{2}(1 - \cos(2t))$$

$$\int \frac{dx}{x} = \ln|x|$$