MATH 002 SAMPLE FINAL (SUPPLEMENTARY)

FOR THE MATERIAL we covered in Chapter 1 please refer your exam 1,exam 2 and exam 3's and their sample tests.

Questions on Section 2.1-2.1-2.4 and 2.5

1) How do you determine a given number is **a** solution to an equation or not? Explain.

2) Below you are given the solution of a student to a linear equation. She has made a mistake and reached a wrong answer. Identify the mistake and explain why it is so. Note that I am **NOT** asking what the correct solution is. I want you to tell her what her mistake is.

$$2x - x - 5 = -x - 1 - 4$$
$$x - 5 = -x - 5$$
$$+5 \qquad +5$$
$$x = -x \quad \Rightarrow \quad \frac{x}{x} = -\frac{x}{x} \Rightarrow 1 = -1$$ Contradiction and No Solution

3) Solve the following linear equations if possible. In the case the equality is an identity or a contradiction makes sure you state which one as well as the solutions. Simplify your answers!!

a) $-3(4t-1) = -(t-14)$

b) $-\frac{5v}{3} = 25$

c) $4y + 3(y-2) = 2(y+4) - (2y-7)$

d) $\frac{q-3}{2} - \frac{4q-1}{6} = \frac{2}{18}$

e) $\frac{6x-10}{5} = \frac{4x+4}{2}$

f) $4(x-1) - x + 6 = 2x - (-x-2)$

g) $5(3u+3) - 10 = 15u + 4$

4) Solve the following literal equations for the variable mentioned.

a) Solve for y:

$$2(3x-y+1)=3(4x-y-2)$$

b) Solve for h_1 :

$$V=\frac{1}{3}\pi r^2(h_1+h_2)$$

c) Solve for R_1 :

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$

5) Fill in the blanks in the following table

Inequality	Number Line Graph	Interval Notation
$x<\pi$		
		$(-3,7]$
		
$5\geq x\geq 0$		

6) Solve the following inequalities if possible. In the case the inequality is an unconditional inequality or a contradiction make sure you state which one as well as the solutions. Write your final answers in interval notation!!!

a) $4(6m+7)-2m \geq 7(3m+1)+1$

b) $\frac{2x}{5}+\frac{1}{2} \geq \frac{3x}{4}+\frac{3}{20}$

c) $-\frac{3}{4}w < -2$

d) $-3(5-2x) > -2(5x+1)+16x$

e) $3x+2 \leq 3(x+1)$

f) $-3 \leq \frac{m-2}{3} < -1$ (Hint: LCD)

g) $5-\frac{x-5}{2} \geq -1$

7) Solve the following quadratic equations either by factoring or by extraction of roots. **Simplify your answers as much as you can.** (Answers containing only these two methods will be accepted. Solutions using other methods will get zero.)

a) $35v^2 = 250$

b) $(5b+1)^2 - 2 = 79$

c) $(r+6)(r-1) = -10$

d) $2w^2 = 7w+15$

e) $\frac{3}{v} = v$ (Hint: The word "proportion" comes to mind!!)

8) Solve each quadratic equation by **completing the square**. **Simplify your answers as much as you can.** Answers that use **only** completing the square method will be accepted. Other solution methods will get zero.

a) $2y^2 - 16y - 112 = 0$

b) $10x = 4x^2$

9) Solve each quadratic equation by **using the Quadratic Formula**. **Simplify your answers as much as you can.** Answers that use **only** the quadratic formula will be accepted. Other solution methods will get zero.

a) $2y^2 - 3y + 8 = y^2 + 7y$

b) $(2x - 3)(x - 5) = 0$

ANSWERS:

1)

You replace x (the variable) with the given # in the eqn. If it makes the eqn. turn into a correct statement then the #, is a sdn.
given

2) $x = -x \Rightarrow \frac{x}{x} = -\frac{x}{x} \Rightarrow 1 = -1$ Contradiction and No Solution

She has divided both sides by "x" without going back & checking whether $x=0$ is a sdn. or not. In her case it is the only solution and she throws it away by dividing both sides by x.

3) a) $-3(4t-1) = -(t-14)$

$-12t+3 = -t+14$

$+12t \quad +12t$

$3 = 11t + 14$

$-14 \quad -14$

$\frac{-11}{11} = \frac{11t}{11}$

$\boxed{-1 = t}$

b) $-\frac{5v}{3} = 25 \Rightarrow 3\left(-\frac{5v}{3}\right) = 25 \cdot 3 \Rightarrow \frac{-5v}{-5} = \frac{75}{-5}$

$\boxed{v = -15}$

c) $4y+3(y-2)=2(y+4)-(2y-7)$

$4y+3y-6=2y+8-2y+7$

$7y-6=15$

$+6 \quad +6$

$\frac{7y}{7}=\frac{21}{7} \Rightarrow \boxed{y=3}$

d) $\frac{q-3}{2}-\frac{4q-1}{6}=\frac{2}{18}$ LCD = 18

$18\left(\frac{q-3}{2}\right)-18\left(\frac{4q-1}{6}\right)=\frac{2}{18}\cdot 18$

$9(q-3)-3(4q-1)=2$

$9q-27-12q+3=2$

$-3q-24=2$

$+24 \quad +24$

$\frac{-3q}{-3}=\frac{26}{-3} \Rightarrow \boxed{q=\frac{26}{-3}}$

e) $\frac{6x-10}{5} = \frac{4x+4}{2}$

$2(6x-10) = 5(4x+4)$

$12x-20 = 20x+20$

$-12x \qquad -12x$

$-20 = 8x+20$

$-20 \qquad -20$

$\frac{-40}{8} = \frac{8x}{8} \Rightarrow \boxed{x=-5}$

f) $4(x-1)-x+6 = 2x-(-x-2)$

$4x-4-x+6 = 2x+x+2$

$3x+2 = 3x+2$

$-2 \qquad -2$

$3x = 3x$

$-3x \quad -3x$

$0=0$ | Identity | Soln: $\mathbb{R}$

g) $5(3u+3)-10 = 15u+4$

$15u+15-10 = 15u+4$

$15u+5 = 15u+4$

$-15u \qquad -15u$

$5=4$ | Contradiction | No Soln.

4) A) Solve for y:

$2(3x-y+1)=3(4x-y-2)$

$6x-2y+2=12x-3y-6$

$+2y \qquad +2y$

$6x+2=12x-y-6$

$-6x \qquad -6x$

$2=6x-y-6$

$+y \qquad +y$

$y+2=6x-6$

$-2 \qquad -2$

$$\boxed{y=6x-8}$$

B) Solve for h_1:

$$V=\frac{1}{3}\pi r^2(h_1+h_2) \Rightarrow 3V=3\left(\frac{1}{3}\pi r^2(h_1+h_2)\right)$$

$$\frac{3V}{\pi r^2}=\frac{\pi r^2(h_1+h_2)}{\pi r^2} \Rightarrow \frac{3V}{\pi r^2}=h_1+h_2$$

$-h_2 \qquad -h_2$

$$\boxed{\frac{3V}{\pi r^2}-h_2=h_1}$$

C) Solve for R_1: LCD$=RR_1R_2$

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2} \Rightarrow RR_1R_2\cdot\frac{1}{R}=RR_1R_2\frac{1}{R_1}+RR_1R_2\frac{1}{R_2}$$

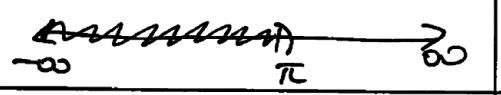
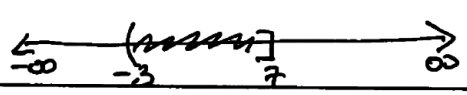
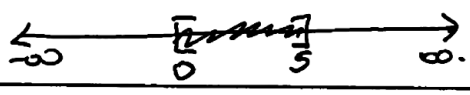
$$R_1R_2=RR_2+RR_1$$

$-RR_1 \qquad -RR_1$

$$R_1R_2-RR_1=RR_2$$

$$\frac{R_1(R_2-R)}{(R_2-R)}=\frac{RR_2}{R_2-R} \Rightarrow \boxed{R_1=\frac{RR_2}{R_2-R}}$$

5) **Fill in the blanks in the following table**

Inequality	Number Line Graph	Interval Notation
$x < \pi$		$(-\infty, \pi)$
$-3 < x \leq 7$		$(-3, 7]$
$5 < x$		$(5, \infty)$
$5 \geq x \geq 0$		$[0, 5]$.

6) a) $4(6m+7) - 2m \geq 7(3m+1) + 1$

$24m + 28 - 2m \geq 21m + 7 + 1$

$22m + 28 \geq 21m + 8$

$-21m \quad -21m$

$m + 28 \geq 8$

$-28 \quad -28$

$m \geq -20$ $\quad [-20, \infty)$

b) $\frac{2x}{5} + \frac{1}{2} \geq \frac{3x}{4} + \frac{3}{20}$ LCD = 20

$4 \cdot 20\left(\frac{2x}{5}\right) + 10 \cdot 20\left(\frac{1}{2}\right) \geq 5 \cdot 20\left(\frac{3x}{4}\right) + 20\left(\frac{3}{20}\right)$

$8x + 10 \geq 15x + 3$

$-8x - 3 \quad -8x - 3$

$7 \geq 7x$

$\frac{7}{7} \quad \frac{7x}{7}$

$1 \geq x$ $\quad (-\infty, 1]$

c) $-\frac{3}{4}w < -2 \Rightarrow$

$4\left(-\frac{3}{4}w\right) < (-2)4$

$-3w < -8$

$\frac{-3w}{-3} \quad \frac{-8}{-3}$

$w > \frac{8}{3}$

$\left(\frac{8}{3}, \infty\right)$

d) $-3(5-2x) > -2(5x+1)+16x$

$-15 + 6x > -10x - 2 + 16x$

$-15 + 6x > 6x - 2$

$-6x \quad -6x$

$-15 > -2$

$+2 \quad +2$

$-13 > 0 \Leftarrow$ contradiction

No Soln.

e) $3x+2 \leq 3(x+1)$

$3x+2 \leq 3x+3$

$-3x \quad -3x$

$2 \leq 3$

Unconditional Ineq.

Soln: $\mathbb{R}$

f) $-3 \le \frac{m-2}{3} < -1$ (Hint: LCD) LCD = 3

$3(-3) \le m-2 \le (-1)3$

$-9 \le m-2 \le -3$

$+2 \quad +2 \quad +2$

$-7 \le m \le -1$ $\quad [-7, -1)$

g) $5 - \frac{x-5}{2} \ge -1$

$-5 \quad -5$

$2\left(-\frac{(x-5)}{2}\right) \ge (-6) \cdot 2$

$-(x-5) \ge -12$

$-x+5 \ge -12$

$-5 \quad -5$

$\frac{-x}{-1} \ge \frac{-17}{-1}$

$x \le 17$ $\quad (-\infty, 17]$

7)

a) $\frac{35v^2}{35} = \frac{250}{35} \Rightarrow v^2 = \frac{50}{7}$

$\sqrt{v^2} = \sqrt{\frac{50}{7}} \Rightarrow |v| = \frac{\sqrt{50}}{\sqrt{7}} = \frac{5\sqrt{2}}{\sqrt{7}}$

$$\boxed{v = \mp \frac{5\sqrt{2}}{\sqrt{7}}}$$

b) $(5b+1)^2 - 2 = 79$

$+2 \quad +2$

$\sqrt{(5b+1)^2} = \sqrt{81} \Rightarrow |5b+1| = 9$

$\Rightarrow (5b+1) = \mp 9$

$-1 \quad -1$

$\frac{5b}{5} = \frac{-1 \mp 9}{5} \Rightarrow b = \frac{-1 \mp 9}{5}$

$b = \frac{-1-9}{5} = \frac{-10}{5}$ $\quad$ $b = \frac{-1+9}{5}$

$\boxed{b = -2}$ $\quad$ $\boxed{b = 8/5}$

c) $(r+6)(r-1) = -10$

$r^2 - r + 6r - 6 = -10$

$r^2 + 5r - 6 = -10$

$+10 \quad +10$

$r^2 + 5r + 4 = 0$

$(r+4)(r+1) = 0$

$r+4 = 0$ $\quad$ $r+1 = 0$

$\boxed{r = -4}$ $\quad$ $\boxed{r = -1}$

$-10 \cdot 3$

$AB = -30$

$A+B = -7$

d) $2w^2 = 7w + 15 \Rightarrow 2w^2 - 7w - 15 = 0$

$(2w^2 - 10w) + (3w - 15) = 0$

$2w(w-5) + 3(w-5) = 0$

$(2w+3)(w-5) = 0$

$2w+3=0 \qquad w-5=0$

$\frac{2w}{2} = \frac{-3}{2} \qquad \boxed{w=5}$ OR

$\boxed{w = -3/2}$

e) $\frac{3}{v} = v$ (Hint: The word "proportion" comes to mind!!)

$\frac{3}{v} \times \frac{v}{1}$

$\sqrt{3} = \sqrt{v^2}$

$\sqrt{3} = |v| \Rightarrow \boxed{v = \mp\sqrt{3}}$

8) a) $2y^2-16y-112=0$

$\frac{2}{2}\ \frac{2}{2}\ \frac{2}{2}\ \frac{2}{2}$

$y^2-8y-56=0$

$y^2-8y=56$ $\quad b=-8 \quad \frac{b}{2}=-4 \quad (-4)^2=16$

$y^2-8y+16=56+16$

$y^2-8y+16=72$

$(y-4)^2=72$

$\sqrt{(y-4)^2}=\sqrt{72}=\sqrt{36\cdot 2}=6\sqrt{2}$

$|y-4|=6\sqrt{2} \Rightarrow y-4=\mp 6\sqrt{2}$

$+4 \quad +4$

$\Rightarrow \boxed{y=4\mp 6\sqrt{2}}$

b) $10x=4x^2 \Rightarrow \frac{4x^2}{4}-\frac{10x}{4}=\frac{0}{4} \Rightarrow x^2-\frac{5}{2}x=0$

$b=-5/2 \quad b/2=-5/4 \quad (-5/4)^2=\frac{25}{16}$

$x^2-\frac{5}{2}x+\frac{25}{16}=0+\frac{25}{16}$

$\sqrt{(x-\frac{5}{4})^2}=\sqrt{\frac{25}{16}}$

$|x-\frac{5}{4}|=\frac{5}{4}$

$x-\frac{5}{4}=\mp\frac{5}{4}$

$+5/4 \quad +5/4$

$x=\frac{5}{4}\mp 5/4$

$x=\frac{5}{4}-\frac{5}{4}$ $\quad x=\frac{5}{4}+\frac{5}{4}$

$\boxed{x=0}$ OR $\boxed{x=\frac{10}{4}=\frac{5}{2}}$ [11]

9) a) $2y^2-3y+8=y^2+7y$

$-y^2-7y \quad -y^2-7y$

$y^2-10y+8=0 \quad a=1 \quad b=-10 \quad c=8$

$$x=\frac{-(-10)\mp\sqrt{(-10)^2-4(1)(8)}}{2(1)}=\frac{10\mp\sqrt{100-32}}{2}$$

$$x=\frac{10\mp\sqrt{68}}{2}=\frac{10\mp\sqrt{4\cdot17}}{2}=\frac{10\mp2\sqrt{17}}{2}=\frac{10}{2}\mp\frac{2\sqrt{17}}{2}$$

$$=\boxed{5\mp\sqrt{17}}$$

b) $(2x-3)(x-5)=0 \Rightarrow 2x^2-10x-3x+15=0$

$$2x^2-13x+15=0$$

$a=2$

$b=-13$

$c=15$

$$x=\frac{-(-13)\mp\sqrt{(-13)^2-4(2)(15)}}{2(2)}$$

$$X=\frac{13\mp\sqrt{169-120}}{4}=\frac{13\mp\sqrt{49}}{4}$$

$$X=\frac{13\mp7}{4}$$

$$x=\frac{13+7}{4} \qquad X=\frac{13-7}{4}$$

$$x=20/4 \qquad X=6/4$$

$$\boxed{X=5} \qquad \boxed{X=3/2}$$