<u>**MATH 002 Exam 3 (FORM A)**</u>

\- Solutions - **Name:**

For factorizations make sure you factor out the GCF first!!!!!

1) **(5pts)** Reduce: $\frac{x-x^2}{x^2-2+x} = \frac{x(1-x)}{x^2+x-2} = \frac{x(1-x)}{(x+2)(x-1)} = \frac{-x\cancel{(x-1)}}{(x+2)\cancel{(x-1)}}$

$= \boxed{\frac{-x}{x+2}}$

2) **(5pts each)** Perform the indicated operation and simplify your results.

a) $\frac{x^2-3x-10}{x^2+2x-35}\cdot\frac{x^2+4x-21}{x^2+9x+14}$

$\frac{\cancel{(x-5)}\cancel{(x+2)}}{\cancel{(x+7)}\cancel{(x-5)}}\cdot\frac{(x-3)\cancel{(x+7)}}{\cancel{(x+2)}(x+7)} = \boxed{\frac{x-3}{x+7}}$

b) $\frac{v}{v-6}\div\frac{2v+4}{v^2-6v}-\frac{3}{v+2} = \frac{v}{\cancel{v-6}}\cdot\frac{v\cancel{(v-6)}}{2(v+2)} - \frac{3}{v+2}$

$= \frac{v^2}{2(v+2)} - \frac{3}{v+2}$ $\quad CD = 2(v+2)$

$= \boxed{\frac{v^2-6}{2(v+2)}}$

c) $\frac{x}{x-3} - \frac{x+1}{x^2+5x-24} = \frac{x}{x-3} - \frac{x+1}{(x+8)(x-3)}$ $\quad CD = (x+8)(x-3)$

$3 \cdot 8$

$$= \frac{x(x+8) - (x+1)}{CD}$$

$$= \frac{x^2+8x-x-1}{CD} = \frac{x^2+7x-1}{CD} = \boxed{\frac{x^2+7x-1}{(x+8)(x-3)}}$$

3) **(5pts)Fill in the missing blanks.**

a) $a^0 =$ 1 if $a \neq$ 0

b) $(a^m)^n =$ a^{mn}

c) $\frac{a^m}{a^n} =$ a^{m-n}

d) $(a \cdot b)^n = a^n \cdot b^n$

e) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

4) **(8pts each) Simplify the following expressions. Make sure your final answers <u>do not contain any</u> <u>negative exponents.</u>**

a) $(5^{-2/5}x^{-4}y^{8})^{-5/4}(5^{-2/5}x^{2}y^{8})^{-5/4}$ $= \left(5^{-\frac{2}{5}-\frac{2}{5}} \; x^{-4+2} \; y^{8+8}\right)^{-5/4}$

$$= \left(5^{-4/5} \; x^{-2} \; y^{16}\right)^{-5/4}$$

$$= 5^{-\frac{4}{5}\cdot\frac{-5}{4}} \; x^{-\cancel{2}(-\frac{5}{\cancel{4}})} \; y^{\cancel{16}^{4}(-5/\cancel{4})}$$

$$= 5^{1} \; x^{5/2} \; y^{-20} = \boxed{\frac{5x^{5/2}}{y^{20}}}$$

b) $\dfrac{\left(27x^{\frac{2}{3}}y^{\frac{2}{3}}\right)^{\frac{1}{3}}}{\left(8x^{\frac{1}{2}}y^{\frac{1}{3}}\right)^{\frac{2}{3}}}$ $= \dfrac{3^{3\cdot 1/3} \; x^{\frac{2}{3}\cdot\frac{1}{3}} \; y^{\frac{2}{3}\cdot\frac{1}{3}}}{2^{3\cdot\frac{2}{3}} \; x^{\frac{1}{2}\cdot\frac{2}{3}} \; y^{\frac{1}{3}\cdot\frac{2}{3}}} = \dfrac{3^{1} \; x^{2/9} \; \cancel{y^{2/9}}}{2^{2} \; x^{1/3} \; \cancel{y^{2/9}}}$

$$= \frac{3x^{\frac{2}{9}-\frac{1}{3}}}{4}$$

$$= \frac{3x^{\frac{2}{9}-\frac{3}{9}}}{4}$$

$$= \frac{3x^{-1/9}}{4}$$

$$= \boxed{\frac{3}{4x^{1/9}}}$$

c) $\dfrac{(25x^2y^3)^{1/3}5^{1/3}x^{1/3}y}{5x^{1/2}y^2} = \dfrac{5^{2\cdot 1/3}x^{2\cdot 1/3}y^{3\cdot 1/3}5^{1/3}x^{1/3}y}{5x^{1/2}y^2}$

$= \dfrac{5^{2/3+1/3}x^{\frac{2}{3}+\frac{1}{3}}y^{1+1}}{5x^{1/2}y^2}$

$= \dfrac{\cancel{5}\,x\,\cancel{y^2}}{\cancel{5}\,x^{1/2}\cancel{y^2}} = x^{1-\frac{1}{2}} = x^{\frac{2}{2}-\frac{1}{2}}$

$= \boxed{x^{1/2}}$

5) (0.5pts each) Fill in the missing form. (Do not evaluate anything just fill in the correct form)

Radical Form	Rational Exponent Form
1) $\sqrt[4]{x^2+y^3}$	$(x^2+y^3)^{\frac{1}{4}}$
2) $\dfrac{1}{\sqrt{x-1}}$	$(x-1)^{-1/2}$
3) $\sqrt[4]{(81)^3}$	$81^{\frac{3}{4}}$
4) $\sqrt[4]{3xy^3}$	$(3xy^3)^{1/4}$

6) **(a-d parts 3pts each) Simplify each radical expression. You may assume variables are <u>non-negative.</u>**

a) $\sqrt{48}$ $= \sqrt{16\cdot 3} = \boxed{4\sqrt{3}}$

b) $\sqrt[3]{-24}$ $= \sqrt[3]{-8\cdot 3} = \boxed{-2\sqrt[3]{3}}$

c) $\sqrt[3]{x^4y^8}$ $= \boxed{xy^2 \sqrt[3]{xy^2}}$

d) $3\sqrt{\frac{3x}{25}}-4\sqrt{\frac{3x}{49}}$ $= \frac{3}{1}\cdot\frac{\sqrt{3x}}{\sqrt{25}} - \frac{4}{1}\frac{\sqrt{3x}}{\sqrt{49}} = \frac{3\sqrt{3x}}{5} - \frac{4\sqrt{3x}}{7}$ CD = 35

$= \frac{21\sqrt{3x}}{35} - \frac{20\sqrt{3x}}{35} = \boxed{\frac{\sqrt{3x}}{35}}$

e) (6pts) $\sqrt[3]{\frac{x^4y^{14}}{x^{25}y^2}}$ $= \sqrt[3]{\frac{y^{14-2}}{x^{25-4}}} = \sqrt[3]{\frac{y^{12}}{x^{21}}} = \frac{\sqrt[3]{y^{12}}}{\sqrt[3]{x^{21}}} = \boxed{\frac{y^4}{x^7}}$

7) (3pts each) Simplify each radical expression where x represents any real number. ***USE absolute value notation*** wherever necessary.

a) $\sqrt{x^{10}} \cdot \sqrt[5]{x^{10}}$ $= \boxed{|x^5|\ x^2}$

b) $\sqrt{32x}\sqrt{2x}$ $= \sqrt{32x \cdot 2x} = \sqrt{64x^2} = \boxed{|8x|}$

8) (3pts each) Rationalize the denominator.

a) $\sqrt[5]{\frac{y^2}{9y^3}}$ $= \sqrt[5]{\frac{1}{9y}} = \frac{\sqrt[5]{1}}{\sqrt[5]{9y}} = \frac{1}{\sqrt[5]{9y}} \cdot \frac{\sqrt[5]{9^4y^4}}{\sqrt[5]{9^4y^4}} = \boxed{\frac{\sqrt[5]{9^4y^4}}{9y.}}$

b) $\frac{5\sqrt{3}}{\sqrt{11}-9}$ $\cdot \frac{\sqrt{11}+9}{\sqrt{11}+9} = \frac{5\sqrt{3}(\sqrt{11}+9)}{(\sqrt{11})^2-9^2}$

$= \frac{5\sqrt{3}(\sqrt{11}+9)}{11-81}$

$= \frac{\cancel{5}\sqrt{3}(\sqrt{11}+9)}{-\cancel{70}}$

$= \boxed{\frac{\sqrt{3}(\sqrt{11}+9)}{-14}}$

9) **Simplify each radical. You may assume all the variables represent positive reals numbers so you <u>don't have to use absolute value</u> symbol.**

a) **(3pts)** $2\sqrt{3}(\sqrt{2}+\sqrt{5})$

$\boxed{2\sqrt{6}+2\sqrt{15}}$

b) **(4pts)** $\left(\sqrt{2x}+\sqrt{y}\right)\left(\sqrt{2x}-5\sqrt{y}\right)$

$= \sqrt{2x}\cdot\sqrt{2x} - 5\sqrt{2x}\sqrt{y} + \sqrt{2x}\sqrt{y} - 5\sqrt{y}\sqrt{y}$

$= 2x - 5\sqrt{2xy} + \sqrt{2xy} - 5y$

$= \boxed{2x - 4\sqrt{2xy} - 5y}$

c) **(4pts)** $5x\sqrt[3]{54x^2} - 2x\sqrt[3]{128x^2}$ $= 5x\cdot 3\sqrt[3]{2x^2} - 2x\cdot 4\sqrt[3]{2x^2}$

$27\cdot 2$ $\quad 64\ 2$

$= 15x\sqrt[3]{2x^2} - 8x\sqrt[3]{2x^2}$

$= \boxed{7x\sqrt[3]{2x^2}}$

d) (4pts) $(2\sqrt{6}-3\sqrt{5})^2$

$(2\sqrt{6}-3\sqrt{5})(2\sqrt{6}-3\sqrt{5})$

$= 4\cdot 6 - 6\sqrt{30} - 6\sqrt{30} + 9\cdot 5$

$= 24 - 12\sqrt{30} + 45$

$= \boxed{69 - 12\sqrt{30}}$

e) (3pts) $3x\sqrt{x^2y} + 7y\sqrt[3]{xy} - 4x\sqrt{x^2y} + 9y\sqrt[3]{xy}$

$3x^2\sqrt{y} + 7y\sqrt[3]{xy} - 4x^2\sqrt{y} + 9y\sqrt[3]{xy}$

$\boxed{-x^2\sqrt{y} + 16y\sqrt[3]{xy}}$