Name: Answers

The following are roughly the instructions for the real exam.

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have three hours to complete this exam. When you are told to stop writing, do it or you will lose all points on the page(s) you write on.
- You may not communicate with other students during this test.
- Keep your eyes on your own paper.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctor.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers when appropriate.
- Before turning in your exam, check to make certain you've answered all the questions.

1. Give an example of each of the following, or say that none exists.
   (a) a rational number that is not real none
   (b) a rational number that is not an integer $\frac{1}{2}$
   (c) a real number that is not rational $\sqrt{2}$
   (d) a whole number that is not a natural number 0

2. Which properties of real numbers are demonstrated by each of the following identities?
   (a) $a+(b+c)=(a+b)+c$ associativity of +
   (b) $0+(b+c)=b+c$ 0 is the additive identity
   (c) $a\cdot(b+c)=ab+ac$ distributivity
   (d) $a+(b+c)=a+(c+b)$ commutativity of +

3. Simplify each of the following.
   (a) $-3^2 = -9$
   (b) $(7+3^2(-2))-4 = -15$
   (c) $8-(3+2\cdot 4)^2 = -113$

4. Simplify each of the following. Do not leave absolute value signs in your answer.
   (a) $|3-4| = 1$
   (b) $\left|2-3|1-3|\right| = 4$
   (c) $|\pi-7| = -\pi+7$
   (d) $|\sqrt{5}-2| = \sqrt{5}-2$
   (e) $|\sqrt{7}-3| = -\sqrt{7}+3$

5. Simplify each of the following.

(a) $\frac{7}{10} + \frac{3}{12} = \frac{19}{20}$

(b) $\frac{11 \cdot 13}{2^5 \cdot 3^2 \cdot 5^4 \cdot 11} + \frac{1}{2^4 \cdot 3^2 \cdot 5^4 \cdot 13} = \frac{19}{260000}$

(c) $\frac{3}{70} - \frac{4}{275} = \frac{109}{2 \cdot 5^2 \cdot 7 \cdot 11}$

(d) $\frac{35}{44} \cdot \frac{66}{14} = \frac{15}{4}$

(e) $\frac{11 \cdot 13}{2^5 \cdot 3^2 \cdot 5^4 \cdot 11} \div \frac{13}{2^4 \cdot 3^2 \cdot 5^4 \cdot 11} = \frac{11}{2}$

6. Which of the following are polynomials? For the ones that are polynomials, state their degree and the number of terms.

   (a) $5x^{800}$ poly, degree 800, 1 term

   (b) 7 poly, degree 0, 1 term

   (c) $5 + x^{1/2}$ not a polynomial

   (d) $\frac{7x+1}{2x}$ not a polynomial

   (e) $(3x^2 + 2)^2$ poly, degree 4, 3 terms

   (f) $1 + x^{-1} + 2x^{-2}$ not a polynomial

7. Simplify the following. (For division, write your answer as a polynomial plus a proper rational expression.)

   (a) $(5x^2 - 3x + 1) - (3x^3 + x - 6)$ $= -3x^3 + 5x^3 - 4x + 7$

   (b) $(3x^2 + x + 4)(2x + 7)$ $= 6x^3 + 23x^2 + 15x + 28$

   (c) $(7x + 2)(7x - 2)$ $= 49x^2 - 4$

   (d) $(x^3 + 3x^2 - 4x + 2) \div (2x^2 - 1)$ $= \frac{1}{2}x + \frac{3}{2} + \frac{-7x+7}{2(2x^2-1)}$

8. Factor the following completely (over the integers, i.e. all your factors should have integer coefficients).

(a) $x^2 + 11x + 30$ $= (x+5)(x+6)$

(b) $x^2 + 13x + 30$ $= (x+3)(x+10)$

(c) $x^2 + 17x + 30$ $= (x+2)(x+15)$

(d) $x^2 - 11x + 30$ $= (x-5)(x-6)$

(e) $2x^3 + 7x^2 - 4x - 14$ $= (2x+7)(x^2-2)$

(f) $2x^3 + 7x^2 - 4x$ $= x(2x-1)(x+4)$

(g) $x^2 - 16$ $= (x+4)(x-4)$

(h) $x^2 + 16$ prime (is already factored completely)

(i) $x^4 - 16$ $= (x-2)(x+2)(x^2+4)$

(j) $x^6 - 1$ $= (x-1)(x+1)(x^2+x+1)(x^2-x+1)$

9. Simplify the following. (For division, write your answer as a simplified rational expression.)

(a) $\dfrac{3x+17}{x^2-5x-6} + \dfrac{x-1}{x^2+x}$ $= \dfrac{4x+6}{x^2-6x}$

(b) $\dfrac{3x+17}{x^2-5x-6} - \dfrac{x-1}{x^2+x}$ $= \dfrac{2x^2+24x-6}{x(x-6)(x+1)}$

(c) $\dfrac{3x+17}{x^2-5x-6} \div \dfrac{x-1}{x^2+x}$ $= \dfrac{x(3x+17)}{(x-6)(x-1)}$

10. Simplify the following. (Some are not real numbers, for which your answer should be "not a real number". All variables represent positive numbers.)

    (a) -3^3 $= -27$

    (b) $(-3)^3 = -27$

    (c) $-27^{1/3} = -3$

    (d) $(-27)^{1/3} = -3$

    (e) $-16^{1/4} = -2$

    (f) $(-16)^{1/4}$ Not a real number

    (g) $(5x^3)^3 = 125x^9$

    (h) $\left(\dfrac{4m^{2/3}n^{-1/3}}{500m^{-1/3}n^{-4/3}}\right)^{-2/3} = \dfrac{25}{m^{2/3}n^{2/3}}$

11. Write $2x^{3/2}$ in radical form. $2\sqrt{x^3}$

12. Write $5\sqrt[4]{7x^6}$ in exponential form.

$5(7x^6)^{1/4}$

13. Find all solutions to the following equations.

(a) $3x - 5 = x + 7$ $x = 6$

(b) $\frac{2}{3}x + \frac{7}{2} = \frac{1}{2}x - \frac{8}{3}$ $x = -37$

(c) $3x - 5 + 2(x + 7) = 5x + 2$ No solutions

(d) $3x - 5 + 2(x + 7) = 5x + 9$ Every real # is a solution

(e) $3x - 5 + 2x(x + 7) = 2x^2 + 15x + 1$ $x = 3$

(f) $\dfrac{3}{2x - 5} = \dfrac{7}{2x - 3}$ $x = \frac{13}{4}$

14. Solve the following literal equations for the indicated variable.

(a) $F = \frac{9}{5}C + 32$ for C $C = \frac{5}{9}(F - 32)$

(b) $V = \ell wh$ for w $w = \dfrac{V}{\ell h}$

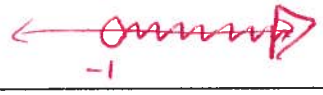
(c) $\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$ for a $a = \dfrac{bc}{b - c}$

(d) $A = \frac{1}{2}h(b_1 + b_2)$ for b_1 $b_1 = \dfrac{2A}{h} - b_2$

15. Find expressions that represent the following.
    (a) the value, in dollars, of n nickels and q quarters $.05n + .25q$
    (b) the value, in cents, of n nickels and $n + 7$ dimes $5n + 10(n+7)$
    (c) the time it takes to travel d miles at v mph d/v
    (d) seven less than three times a number x $3x-7$
    (e) A certain class is broken into groups of four, two girls and two boys in each group, except that the last group is all girls. Give an expression for the number of groups if there are g girls total. $\frac{g-2}{2}$ n

16. Set up and solve appropriate equation(s) to answer the following questions.
    (a) You spend \$1.10 on a hammer and a nail. The hammer cost \$1 more than the nail. How much did each item cost? 5¢ nail, \$1.05 hammer
    (b) One number is 12 less than another, and their sum is 40. Find those numbers. 26 & 14
    (c) One number is 12 less than another, and their sum is 23. Find those numbers. 17.5 & 5.5
    (d) The sum of three consecutive integers is 36. What are those integers? 11, 12, 13
    (e) A car and a motorcycle leave Champaign at noon, heading in opposite directions. The motorcycle travels 5mph faster than the car. They are 387 miles apart at 3pm. What were their speeds? car: 62 mph & motorcycle: 67 mph
    (f) \$11000 is divided between a savings and a checking account. The checking account has \$2000 more than half the savings account. How much is in each account?
    savings: \$6000, checking: \$5000

17. Complete the following table.

inequality	interval	graph
$x > -1$	$(-1, \infty)$	
$x < -5$	$(-\infty, -5)$	
$-4 < x \leq 6$	$(-4, 6]$	

18. Solve the following inequalities. Give your answers in each of the following forms: **1.** a simplified inequality, **2.** interval notation, **3.** a graph on the real line.

(a) $-5 < 3 - 2x \leq 9$ $-3 \leq x < 4$ $[-3, 4)$ 

(b) $3x + 2 \leq 3x + 7$ $2 \leq 7$ $(-\infty, \infty)$ 

(c) $3x + 2 \leq 5x + 7$ $x \geq -\frac{5}{2}$ $[-\frac{5}{2}, \infty)$ 

(d) $5x + 2 \leq 5x - 2$ $2 \leq -2$ $\emptyset$ 

(e) $2x + 9 < 3x + 2 \leq 2x + 7$

$7 < x \leq 5$ $\emptyset$ 

19. Find all (real) solutions to the following equations by the method indicated.

(a) $x^2 - 2x - 8 = 0$ by factoring $x = -2$ or $x = 4$

(b) $3x^2 + 12x - 4 = 0$ by completing the square $x = -2 \pm \frac{4}{\sqrt{3}}$

(c) $3x^2 + 11x + 4 = 0$ using the quadratic formula $x = \frac{-11 \pm \sqrt{73}}{6}$

20. Use the discriminant to determine how many real solutions the following equations have.

(a) $x^2 + 5x - 2 = 0$ 2

(b) $x^2 + 5x + 2 = 0$ 2

(c) $4x^2 + 4x = -1$ 1

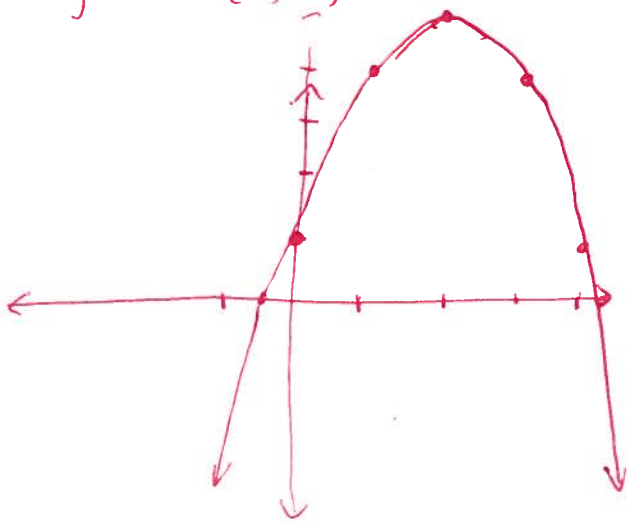
(d) $7x^2 - 1 = 4x$ 2

21. For each of the following equations, find (and clearly label) the x- and y-intercepts, plot at least five points, then sketch the plot.

(a) $y + (x-2)^2 = 5$

x-ints: $(2-\sqrt{5}, 0)$ & $(2+\sqrt{5}, 0)$

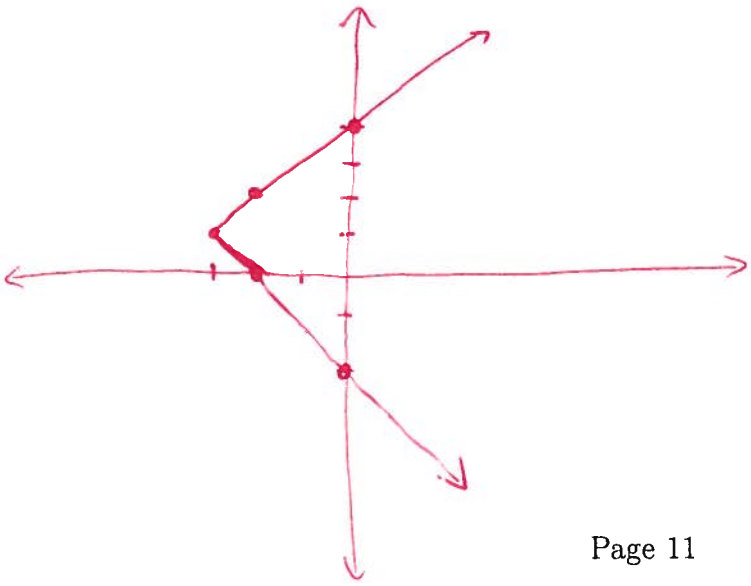
y-int: $(0, 1)$



(b) $x = |y-1| - 3$

x-int: $(-2, 0)$

y-ints: $(0, -2)$ & $(0, 4)$



22. How many points on the y-axis are the same distance from $(1,2)$ as from $(3,4)$? Find all such points. 1 : $(0,5)$

23. How many points on the y-axis are the same distance from $(1,2)$ as from $(3,2)$? Find all such points. none

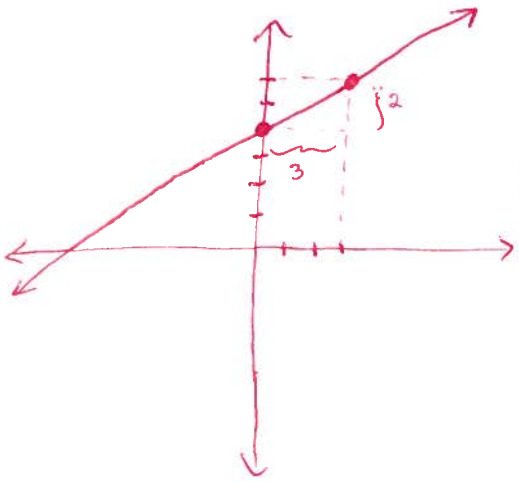
24. How many points on the y-axis are at distance 5 from $(1,2)$? Find all such points.

2 : $(0, 2 \pm 2\sqrt{6})$

25. Identify the slope of the line $y = 4 + \frac{2}{3}x$ and use this to graph the line. slope $= \frac{2}{3}$

26. Find the equation of the line that is perpendicular to the line from Problem (25) and passes through the point $(3, -1)$.

$y + 1 = -\frac{3}{2}(x - 3)$



27. The price of widgets in 1997 was \$1000 per crate. The price of widgets in 2005 was \$2400 per crate. Write these data as two points (x, y) where x represents the year and y represents the cost per crate of widgets. Assuming the cost of widgets grows linearly, **identify and interpret** the slope (treating y as a linear function of x).

(1997, 1000) & (2005, 2400)

slope = 175

Every year, the price of a crate of widgets increased by \$175.

28. Find equations for each of the following lines.

(a) with y-intercept $(0, 2)$ and slope $-\frac{3}{2}$ $y = -\frac{3}{2}x + 2$

(b) with x-intercept $(-1, 0)$ and slope $\frac{1}{2}$ $y = \frac{1}{2}(x+1)$

(c) parallel to the line $2x + 3y = 5$ and passing through $(-1, 4)$ $y - 4 = -\frac{2}{3}(x+1)$

(d) with the same x-intercept as $x + y = 5$ and the same y-intercept as $x - 2y = 1$ $y = \frac{1}{10}x - \frac{1}{2}$

(e) that is horizontal and passes through $(2, 6)$ $y = 6$

(f) the perpendicular bisector of the segment joining $(-3, 2)$ to $(1, 5)$ (the perpendicular bisector is perpendicular to the segment and passes through its midpoint)

$y - \frac{7}{2} = \frac{3}{4}(x+1)$

29. Which of the following are functions?
    (a) Assign to each rectangle its area. ✓
    (b) Assign to each positive real number x the side length of a rectangle with area x. ✗
    (c) Assign to each positive real number x the side length of a square with area x. ✓
    (d) $f(x) = x^2$ ✓
    (e) $f(x) = \sqrt{x}$ ✓
    (f) $f(x) = \pm\sqrt{x}$ ✗

30. Let $g(x) = x^2 - 5x$. Find and simplify the following.
    (a) $g(-5)$ = 50

    (b) $-g(5)$ = 0

    (c) $g(2x)$ = $4x^2 - 10x$

    (d) $2g(x)$ = $2x^2 - 10x$

    (e) $g(x+h)$ = $x^2 + 2xh + h^2 - 5x - 5h$

    (f) $g(x) + g(h)$ $x^2 + h^2 - 5x - 5h$

    (g) $\dfrac{g(x+h) - g(x)}{h}$ = $2x + h - 5$