Name: Solutions

The following are roughly the instructions for the real exam.

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have two hours to complete this exam. When you are told to stop writing, do it or you will lose all points on the page(s) you write on.
- You may not communicate with other students during this test.
- Keep your eyes on your own paper.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctor.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers when appropriate.
- Before turning in your exam, check to make certain you've answered all the questions.

1. Place an 'x' in a cell below if (and only if) the number for that row belongs in the set for that column.

	$\mathbb{N}$	$\mathbb{W}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$	
$1/5$				X	X	
$3 = 3/1$	X	X	X	X	X	
$-7 = -7/1$			X	X	X	
$-3/4$				X	X	
$0 = 0/2$		X	X	X	X	
$2/0$						can't divide by 0
$3 = \sqrt{9}$	X	X	X	X	X	
$\sqrt{3}$					X	
3^2	X	X	X	X	X	
$\sqrt{-9}$						Imaginary number
$-\sqrt{9}$			X	X	X	

2. Which properties of real numbers are demonstrated by each of the following equalities?

(a) $1(\pi x) = (1\pi)x$ Associativity of multiplication

(b) $1(\pi x) = \pi x$ 1 is the multiplicative identity

(c) $1(\pi + x) = (1\pi) + (1x)$ Distributivity

(d) $1(\pi x) = 1(x \cdot \pi)$ Commutativity of multiplication

(e) $1(0 + x) = 1x$ 0 is the additive identity

3. Simplify $7(3 + 2(3 - 2^2))$.

$= 7(3+2(3-4))$

$= 7(3+2\cdot(-1))$

$= 7(3-2)$

$= 7 \cdot 1$

$= 7.$

4. Convert the following to decimal.

(a) $\frac{3}{40}$

0.075

$40\overline{)3.000}$

280

200

or: $= \frac{3}{4} \cdot \frac{1}{10}$

$= 0.75 \cdot \frac{1}{10}$

$= 0.075$

(b) $\frac{1}{6}$

$0.1\overline{6}$

$6\overline{)1.00}$

6

40

36 ← same remainder

4

$0.1\overline{6}$

5. Convert each of the following to a fraction in least terms.

(a) $0.\overline{162} = x$ (2)

$162.\overline{162} = 1000x$ (1)

(1) − (2): $999x = 162$

$x = \frac{162}{999} \overset{\div 9}{=} \frac{18}{111} \overset{\div 3}{=} \boxed{\frac{6}{37}}$

(b) $0.0245 = \frac{245}{10000} \overset{\div 5}{=} \boxed{\frac{49}{2000}}$ ($49 = 7^2$, 7 is not a factor of 2000)

(c) $0.0\overline{1} = x$

$0.1\overline{1} = 10x$

$9x = 0.1$

$90x = 1$

$x = \boxed{\frac{1}{90}}$

6. Find the least common multiple and greatest common divisor for each of the following sets of numbers. You may leave answers as products. Be sure to clearly label which is the lcm and which is the gcd!

(a) 28, 50

$28 = 2^2 \cdot 7$
$50 = 2 \cdot 5^2$
$\text{lcm} = 2^2 \cdot 5^2 \cdot 7$
$\text{gcd} = 2$

(b) 2,4,7

$2 = 2$
$4 = 2^2$
$7 = 7$
$\text{lcm} = 2^2 \cdot 7$
$\text{gcd} = 1 \quad (= 2^0 \cdot 7^0)$

(c) 2,4,12

$2 = 2$
$4 = 2^2$
$12 = 2^2 \cdot 3$
$\text{lcm} = 2^2 \cdot 3$
$\text{gcd} = 2$

(d) 8, 12, 21

$8 = 2^3$
$12 = 2^2 \cdot 3$
$21 = 3 \cdot 7$
$\text{lcm} = 2^3 \cdot 3 \cdot 7$
$\text{gcd} = 1$

(e) 6, 7, 15, 21, 28

$6 = 2 \cdot 3$
$7 = 7$
$15 = 3 \cdot 5$
$21 = 3 \cdot 7$
$28 = 2^2 \cdot 7$
$\text{lcm} = 2^2 \cdot 3 \cdot 5 \cdot 7$
$\text{gcd} = 1$

7. Simplify the following. (Write them as one fraction in lowest terms.)

(a) $\frac{7}{12}+\frac{3}{14}$ $= \frac{7}{12}\cdot\frac{7}{7}+\frac{3}{14}\cdot\frac{2\cdot3}{2\cdot3}$

$12 = 2^2\cdot 3$

$14 = 2 \cdot 7$

$lcm = 2^2\cdot 3\cdot 7$

$= \frac{49}{lcm}+\frac{18}{lcm} = \frac{49+18}{lcm} = \boxed{\frac{67}{84}}$ ← prime

(b) $\frac{7}{12}-\frac{3}{14}$ $= \frac{49}{84}-\frac{18}{84}$

$= \boxed{\frac{31}{84}}$ ← prime

(c) $\frac{\cancel{7}^{1}}{\cancel{12}_{4}}\cdot\frac{\cancel{3}^{1}}{\cancel{14}_{2}} = \frac{1}{4}\cdot\frac{1}{2} = \boxed{\frac{1}{8}}$

(d) $\frac{7}{12}\div\frac{3}{14}$ $= \frac{7}{\cancel{12}_{6}}\cdot\frac{\cancel{14}^{7}}{3} = \boxed{\frac{49}{18}}$

↑ fully reduced

8. Expand and simplify the following. (Write them in standard form.)

(a) $(3x^2 + 2x - 1) + (x^3 - 5x + 7)$

$= x^3 + 3x^2 - 3x + 6$

(b) $(x^3 - 3x + 7) - (3x^2 + 5x + 6)$

$= x^3 - 3x^2 - 8x + 1$

(c) $(x^2 + 1)(x - 3)$

$= x^3 - 3x^2 + x - 3$

(d) $(x + y)(x - y)$

$= x^2 - y^2$

(e) $(x - 1)^2 - (x + 3)^2$

$= (x^2 - 2x + 1) - (x^2 + 6x + 9)$

$= -8x - 8$

OR

$= ((x-1) + (x+3))((x-1) - (x+3))$

$= (2x + 2)(-4)$

$= -8x - 8$

9. Compute the following. (Write the answer as a polynomial plus a "proper" rational expression.)

(a) $(x^2 + 5x - 14) \div (x - 2)$

$$
\begin{array}{r|l}
x-2 & x^2+5x-14 \\
 & -(x^2-2x) \\
 & \quad 7x-14 \\
 & \quad -(7x-14) \\
 & \quad\quad 0
\end{array}
\qquad x+7
$$

(b) $(x^4 - 4x^3 - 7x^2 + 10x) \div (x - 1)$

$$
\begin{array}{r|l}
x-1 & x^4-4x^3-7x^2+10x \\
 & -(x^4-x^3) \\
 & \quad -3x^3-7x^2+10x \\
 & \quad -(-3x^3+3x^2) \\
 & \quad\quad -10x^2+10x \\
 & \quad\quad -(-10x^2+10x) \\
 & \quad\quad\quad 0
\end{array}
\qquad x^3-3x^2-10x
$$

(c) $(x^4 - 3x^3 + x - 7) \div (x^2 + 1)$

$$
\begin{array}{r|l}
x^2+1 & x^4-3x^3+0x^2+x-7 \\
 & -(x^4+1x^2) \\
 & \quad -3x^3-x^2+x-7 \\
 & \quad -(-3x^3-3x) \\
 & \quad\quad -x^2+4x-7 \\
 & \quad\quad -(-x^2-1) \\
 & \quad\quad\quad 4x-6
\end{array}
\qquad x^2-3x-1
$$

$$x^2-3x-1+\frac{4x-6}{x^2+1}$$

10. Completely factor the following. (Work over the integers, i.e. don't get factors with non-integer coefficients.) Problems continue to the next page.

(a) $x^4 - 4x^3 - 7x^2 + 10x$ (hint: see problem 9(b) to get started)

$= (x^3 - 3x^2 - 10x)(x-1)$

$= x(x^2 - 3x - 10)(x-1)$

$= x(x-5)(x+2)(x-1)$

(b) $x^3 - 3x^2y + xy - 3y^2$

$= x^2(x-3y) + y(x-3y)$

$= (x^2+y)(x-3y)$

(c) $x^3 - 8 = (x)^3 - (2)^3$

$= (x-2)(x^2 + 2x + 2^2)$

$= (x-2)(x^2+2x+4)$

prime

(there are not two integers whose product is 4 & whose sum is 2)

(d) $3x^2 + 13x - 10$

$A \cdot B = -30$, $A + B = 13$ $\Rightarrow -2, 15$

$= 3x^2 - 2x + 15x - 10$

$= x(3x-2) + 5(3x-2)$

$= (3x-2)(x+5)$

(e) $2x^2 - 3x + 7$

$A \cdot B = 14$

$A + B = -3$

$-1 + -14 = -15$ X

$-2 + -7 = -9$ X

Prime!

(f) $x^4 - 9x^2 + 20$

Let $y = x^2$, so

$= y^2 - 9y + 20$

$A \cdot B = 20$, $A + B = -9$ $\Rightarrow -4, -5$

$= y^2 - 4y - 5y + 20$

$= y(y-4) - 5(y-4)$

$= (y-4)(y-5)$

$= (x^2-4)(x^2-5)$

$= (x+2)(x-2)(x^2-5)$

difference of squares

prime over $\mathbb{Z}$

11. Simplify the following. (Assume all denominators are nonzero.)

(a) $\dfrac{4-y}{2y^2-7y-4}$ $= \dfrac{4-y}{(y-4)(2y+1)} = \dfrac{-(y-4)}{(y-4)(2y+1)} = \boxed{\dfrac{-1}{2y+1}}$

$A \cdot B = -8$
$A + B = -7$
$\Rightarrow -8, +1$

$2y^2 - 8y + y - 4$
$= 2y(y-4) + 1(y-4)$
$= (y-4)(2y+1)$

(b) $(8a^2-16a) \cdot \dfrac{a-4}{a^3-16a}$ $= \dfrac{8a(a-2)}{1} \cdot \dfrac{(a-4)}{a(a+4)(a-4)} = \boxed{\dfrac{8(a-2)}{a+4}}$

$8a(a-2)$

$a(a^2-16)$
$= a(a+4)(a-4)$

(c) $\dfrac{7}{x^2-5x-6} - \dfrac{12}{x^2-36}$

$(x-6)(x+1)$ $(x-6)(x+6)$

$= \dfrac{7}{(x-6)(x+1)} \cdot \dfrac{x+6}{x+6} - \dfrac{12}{(x-6)(x+6)} \cdot \dfrac{x+1}{x+1}$

$LCM = (x+1)(x-6)(x+6)$

$= \dfrac{7(x+6)}{LCM} - \dfrac{12(x+1)}{LCM}$

$= \dfrac{(7x+42) - (12x+12)}{LCM}$

$= \dfrac{-5x+30}{LCM} = \dfrac{-5(x-6)}{(x+1)(x-6)(x+6)}$

$= \boxed{\dfrac{-5}{(x+1)(x+6)}}$