Name: _____

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have until 10:00pm to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.
- You do not need to simplify algebraic expressions.
- Be sure to check whether a problem asks you to *compute* or just *set up* an integral.
- When you apply a theorem, say so.

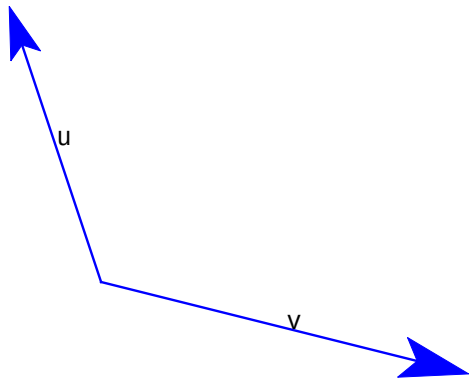
Some possibly useful formulas:

$$\cos^2 t = \frac{1}{2}(1 + \cos(2t))$$
$$\sin^2 t = \frac{1}{2}(1 - \cos(2t))$$
$$\frac{d}{dt} \arctan t = \frac{1}{1+t^2}$$

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	6	11	10	8	12	12	17	8	10	26	120
Score:											

1. (6 points) Let $f(x,y) = xy^2 + x^2 + y^2 - \frac{1}{3}x^3 + 1$. Find and classify all local extrema of f. Does it have global (absolute) extrema?

2. Here are two vectors $\vec{u}$ and $\vec{v}$.



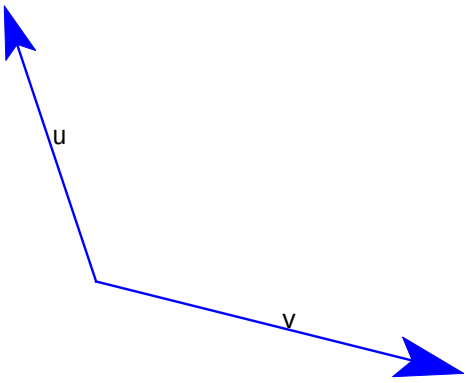
(a) (2 points) Add to the picture above the sum $\vec{u} + 2\vec{v}$.

(b) (2 points) Is the dot product $\vec{u} \cdot \vec{v}$ positive, negative, or zero? How do you know?

(c) (2 points) Treating $\vec{u}$ and $\vec{v}$ as 3D vectors that live in the plane of this paper, which direction is $\vec{u} \times \vec{v}$?

(d) (2 points) What is the geometric interpretation of $||\vec{u} \times \vec{v}||$?

(e) (3 points) Give the formula for, and indicate in the picture below, the projection of $\vec{u}$ in the direction of $\vec{v}$.



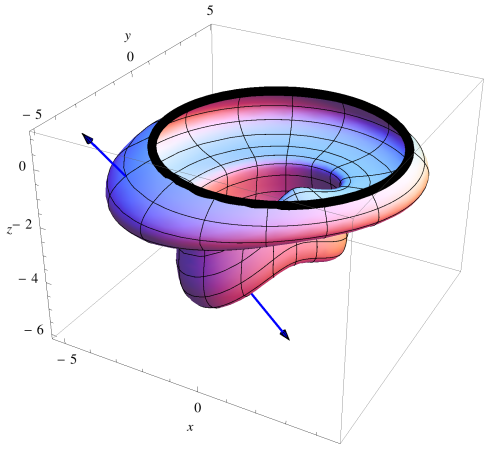
3. Set up (using any valid method) $\iiint_R x^2 z \, dV$ for the following regions R:

   (a) (5 points) the solid bounded above by the inverted cone $z = 4 - \sqrt{x^2 + y^2}$ and below by the xy-plane.

   (b) (5 points) the solid bounded below by the xy-plane, on the sides by the sphere of radius 2, and above by the cone $z = \sqrt{x^2 + y^2}$.

4. (8 points) Set up an integral (that you could input into Mathematica) that measures the flow of $\vec{F}(x, y, z) = \langle x^2, y^2, z^2 \rangle$ across the part of the surface $z = y^2 - x^2$ with $x^2 + y^2 \leq 1$. (To be input into Mathematica, there should be no undefined symbols aside from the variables of integration.)

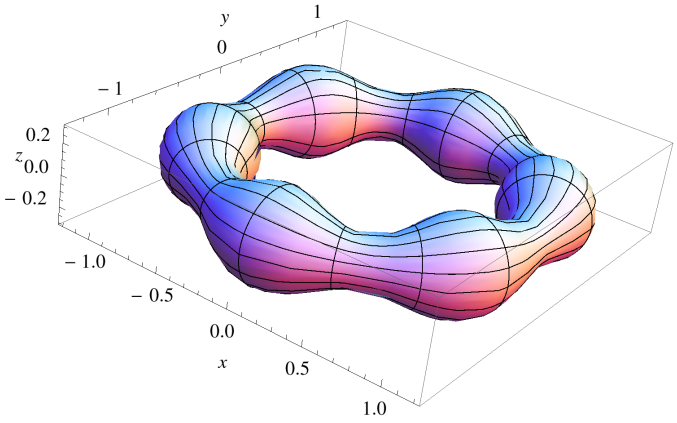
5. The surface R is shown below; its boundary is the circle of radius 2 in the xy-plane. Let $\vec{F}(x, y, z) = \langle x, y + z^2, y - z \rangle$.



(a) (6 points) Find the direction of the flow of $\vec{F}$ across R. (Is it in the direction of the displayed normal vectors or opposite?

(b) (6 points) Find the net flow (amount and direction) of curl $\vec{F}$ across R.

6. The surface S is shown below; it has no boundary. Let $\vec{F}(x, y, z) = \langle x, y + z^2, y - z \rangle$.



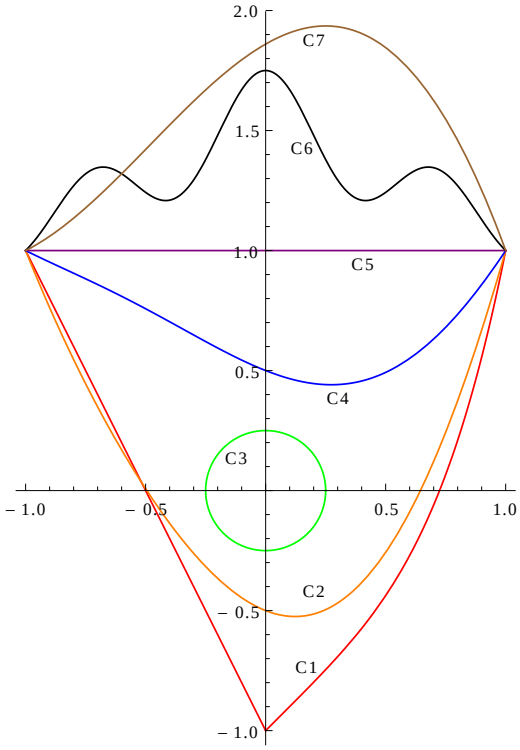
(a) (6 points) Find the direction of the flow of $\vec{F}$ across S. (Is it inward or outward?)

(b) (6 points) Find the net flow (amount and direction) of curl $\vec{F}$ across S.

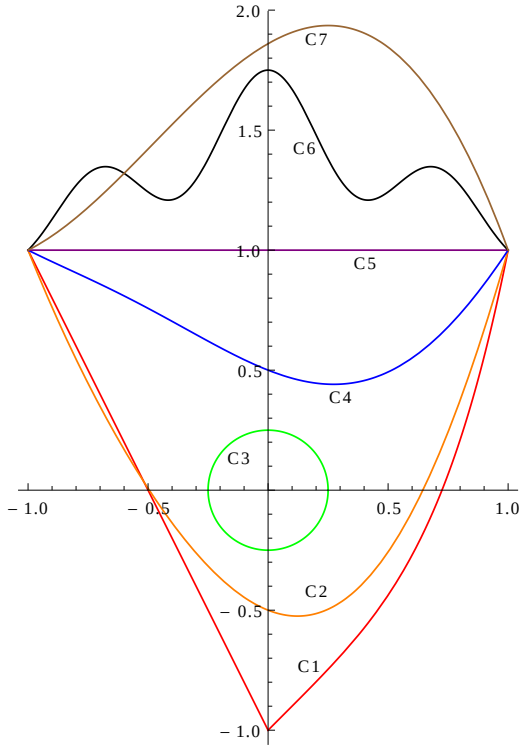
7. Below are shown several curves in the plane. Consider $\vec{F}(x,y) = \left\langle \dfrac{x}{x^2+y^2}, \dfrac{y}{x^2+y^2} \right\rangle$.

(a) (2 points) Compute $\operatorname{div} \vec{F}$.

(b) (2 points) Compute $\operatorname{rot} \vec{F}$.



(c) (6 points) Compute directly the flow of $\vec{F}$ across C_5. (Which direction is it?)



(d) (5 points) Without further computation, what can you say about the flow of $\vec{F}$ across each of the other curves?

(e) (2 points) If I tell you further that $\int_{C_3} \vec{F} \cdot \langle dy, -dx \rangle = 2\pi$, where C_3 is parametrized counterclockwise, then can you improve your statements from (d)?

8. (8 points) Consider the two lines

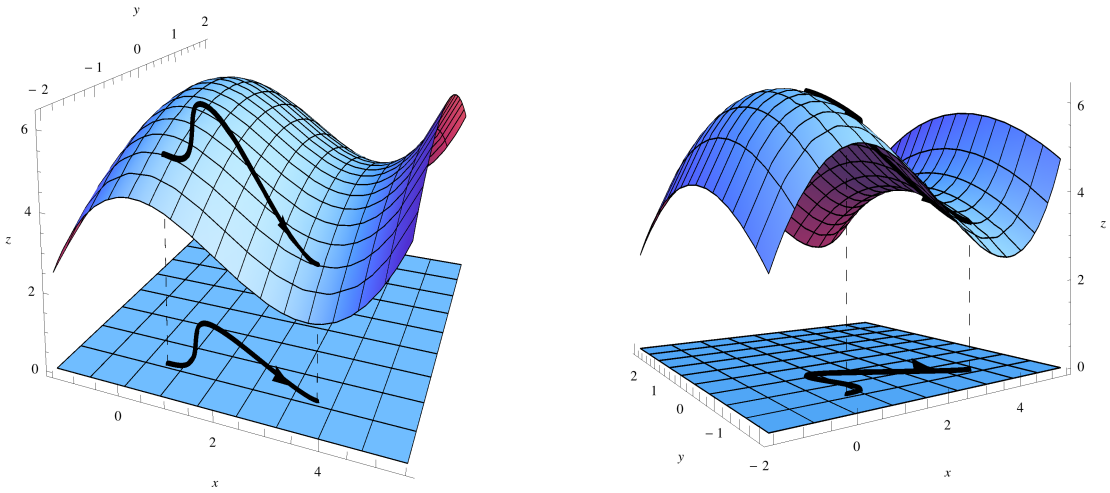
$$\ell_1(t) = \langle 1, 2, 3\rangle + t\langle -3, 1, -2\rangle;$$
$$\ell_2(t) = \langle 4, 1, 8\rangle + t\langle 3, -1, 4\rangle.$$

If these lines intersect or are parallel, find the plane containing them both. Otherwise, find the distance between them.

9. (a) (8 points) Find the maximum and minimum values of $f(x, y) = xy$ on the elliptic disk $x^2 + 2y^2 \leq 1$.

(b) (2 points) Sketch the region of interest together with a few level curves of f, including those level curves corresponding to your maximum and minimum values.

10. Below is shown the graph of a function $f(x, y)$ over the solid rectangle R with $-1.5 \leq x \leq 5.5$, $-2 \leq y \leq 2$, $z = 0$. Also shown is a curve C in the xy-plane together with its "lift" to the graph. When asked to estimate, use exact formulas where possible then give estimates for the expressions in those formulas.

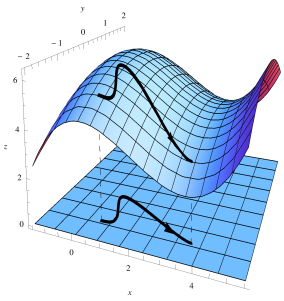


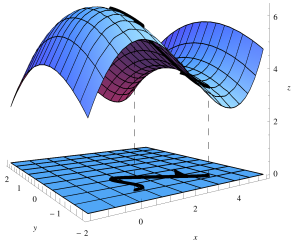
(a) (5 points) Compute/estimate $\displaystyle\int_C \nabla f \cdot \vec{dr}$.

(b) (5 points) Locate (approximately) and classify the critical points of f on the interior of R.

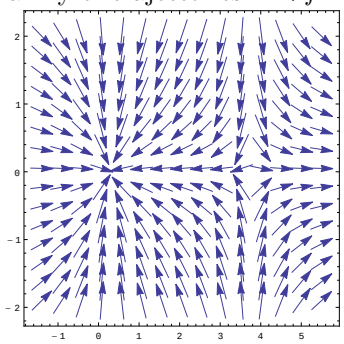
(c) (5 points) Compute/estimate $\displaystyle\iint_R f\, dA$.

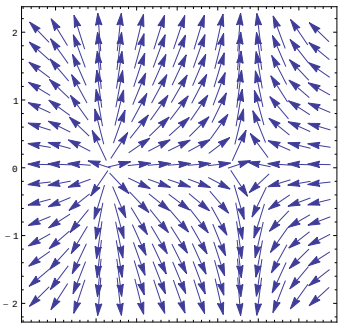
(d) (5 points) Compute/estimate $\displaystyle\int_{C_2} \nabla f \cdot \vec{dr}$, where C_2 is the boundary of R.

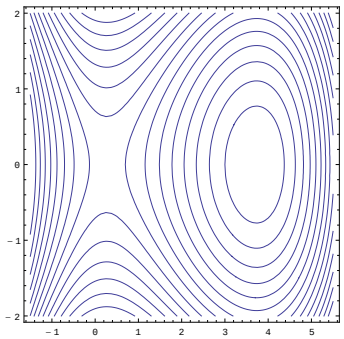


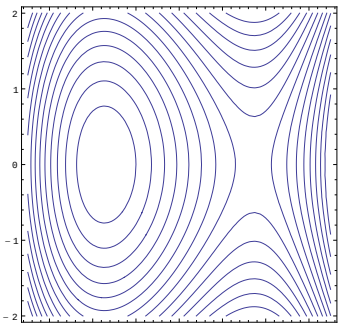


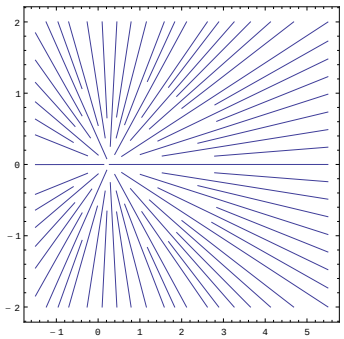
(e) (6 points) Which of the following six images is the contour map of f? Which is the plot of ∇f? Which is the family of trajectories in ∇f?

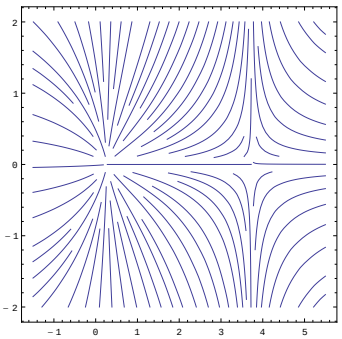












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