Name: ____

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have until 12:50 to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Sign below these instructions to indicate that you have read and agree to them.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.
- You do not need to simplify algebraic expressions.

Some possibly useful formulas:

$$\cos^2 t = \frac{1}{2}(1+\cos(2t))$$
$$\sin^2 t = \frac{1}{2}(1-\cos(2t))$$

Question:	1	2	3	4	5	6	7	8	Total
Points:	6	12	24	18	12	8	10	10	100
Score:									

1. (6 points) Give the formulas for the centroid of a region R in the plane.

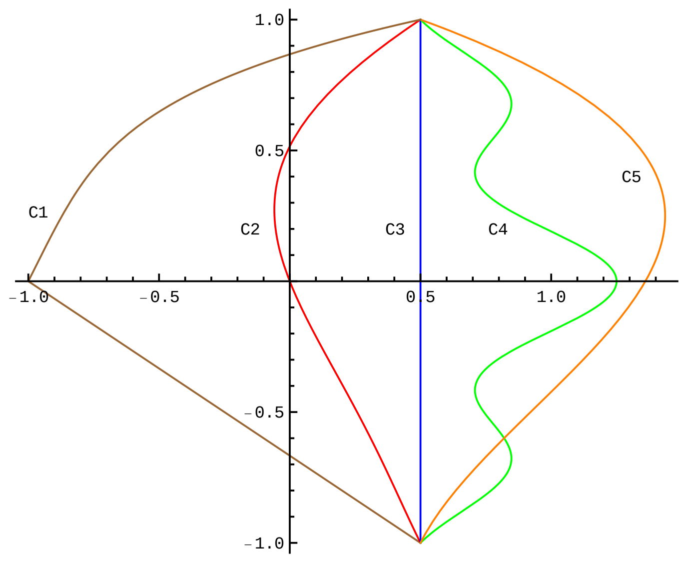
2. (12 points) Compute $\displaystyle\iint_R x^2\,dA$, where R is the region in the plane bounded by the lines $y = x$, $x = 2$, and $y = 6 - x$.

3. Consider the vector field $\vec{F}(x,y) = \left\langle \frac{x-1}{(x-1)^2+y^2}, \frac{y}{(x-1)^2+y^2} \right\rangle$.

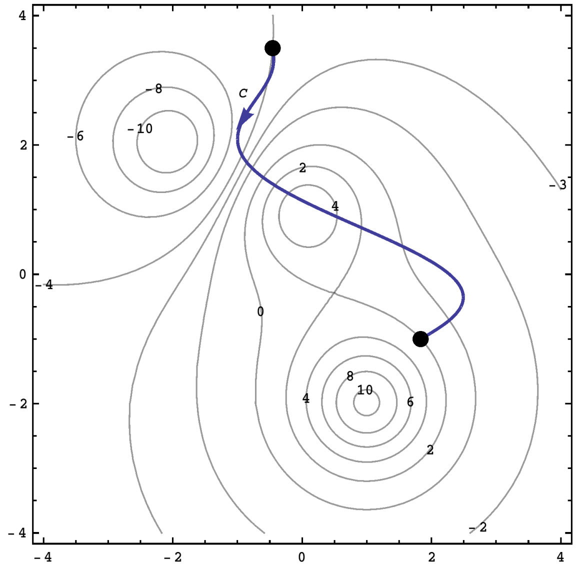
(a) (6 points) Which part(s) of the gradient test does $\vec{F}$ pass?

(b) (10 points) Directly compute the flow of $\vec{F}$ along C_3 shown below.

(c) (8 points) Say as much as you can about the flow on each of the other curves shown.



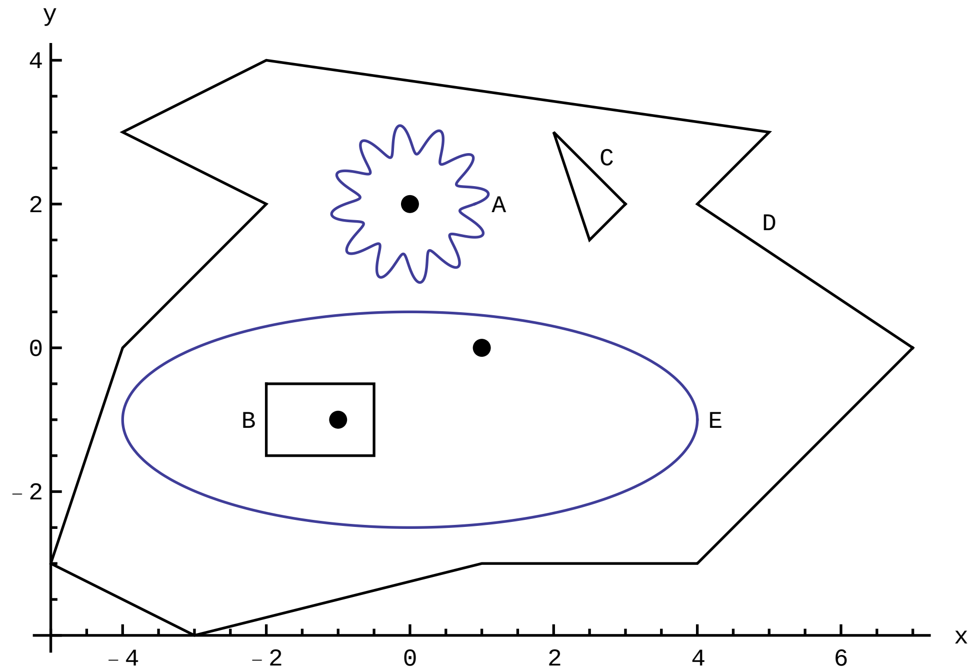
4. Below is shown the contour map of a function f, together with a curve C.



(a) (8 points) Find $\int_C \nabla f \cdot \langle dx, dy \rangle$. (Note that the orientation of C is given in the picture.)

(b) (10 points) Estimate $\iint_R f \, dA$, where R is the solid rectangle with vertices $(-3,1), (-3,3), (-2,3), (-2,1)$.

5. (12 points) A certain vector field $\vec{F}$ has $\operatorname{rot} \vec{F} = 1$ and $\operatorname{div} \vec{F} = 0$ everywhere except at its three singularities at $(-1,-1)$, $(0,2)$, and $(1,0)$. Below are shown several curves as well as the three singularities of $\vec{F}$.



You are given the following pieces of information:

$$\int_A \vec{F} \cdot \langle dy, -dx \rangle = 4, \qquad \int_D \vec{F} \cdot \langle dy, -dx \rangle = 7.$$

(Assume all curves are parametrized counterclockwise.) Say as much as possible about the following. Give brief explanations.

(a) net flow of $\vec{F}$ along C

(b) net flow of $\vec{F}$ across C

(c) net flow of $\vec{F}$ across E

(d) net flow of $\vec{F}$ across B

6. (8 points) Find all sources and sinks of the vector field $\vec{F}(x, y) = \langle x^4, x^2y^2 \rangle$

7. (10 points) Compute the area of the region in the first quadrant bounded by $y = x^2$, $y = x^2 + 1$, $y = 6 - x^2$, and $y = 9 - x^2$. You may stop when you have a double integral over a rectangle.

8. (10 points) Compute $\iint_D e^{x^2+y^2}\,dx\,dy$, where D is the quarter of the disk of radius 2 that lies in the first quadrant. That is, D is defined by the inequalities $x^2+y^2 \leq 4$, $x \geq 0$, and $y \geq 0$.

Scratch Paper - you may remove this if you find it convenient

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