WORKSHOP 20: §9.7, 9.9 MARCH 30, 2017

Name: ______________________________

(1) Let R be a relation on a set B, and let $A \subset B$. The *restriction of R to A* is defined as the relation $S = \{(x, y) \in R : x \in A \text{ and } y \in A\}$ on the set A. [That is, we just ignore all the elements of $B - A$.] Prove that:
   - (a) If R is reflexive, then so is the restriction to A.
   - (b) If R is antireflexive, then so is the restriction to A.
   - (c) If R is symmetric, then so is the restriction to A.
   - (d) If R is antisymmetric, then so is the restriction to A.
   - (e) If R is transitive, then so is the restriction to A. [This is the only vaguely tricky one.]

   Note that this implies that if R is a partial order, then so is the restriction to A, and if R is an equivalence relation, then so is the restriction to A.

(2) **Theorem.** *Every nonempty finite poset has a maximal element.*
   - (a) The proof is by induction. What should be the predicate P so that $\forall n P(n)$ is equivalent to the given theorem? *(Hint: it will still need a universal quantifier.)*
   - (b) Prove $P(n)$ for a few small n. One or more of these will make up the base case.
   - (c) Prove the inductive step: assume for some arbitrary integer $k \geq 1$ that $P(k)$ is true. Your goal is to prove that $P(k + 1)$ is true. How do you prove that statement? How can you make use of $P(k)$? *(That might be a little tough. Struggle a little, but not forever, before asking for hints.)*

(3) Give an example to show that the previous theorem need not be true for infinite posets.

(4) Draw the Hasse diagram for $(\{1, 2, 6, 7, 11, 12, 14, 35, 42\}, |)$. What are the maximal/minimal elements?

(5) Which of the following are equivalence relations? For those that are, give their equivalence classes (explicitly if possible, otherwise just describe them).
   - (a) on the set of classes, xRy iff they have the same subject code
   - (b) on the set of classes, xRy iff they share subject code or course number
   - (c) on $\mathcal{P}(\mathbb{Z})$, ARB iff $|A| = |B|$
   - (d) on $\mathbb{Z}^+$, xRy iff they share a prime factor
   - (e) on $\mathbb{Z}^+$, xRy iff the sets of their prime factors are equal
   - (f) on $\mathbb{Z}^+$, xRy iff the sets of their prime factors have the same cardinality

(6) Consider the relation R on binary strings of length 3, defined by xRy if and only if y can be obtained from x by changing some number of 0's to 1's.
   - (a) Verify that R is a partial order.
   - (b) Draw the Hasse diagram for R. It should look familiar. Why?