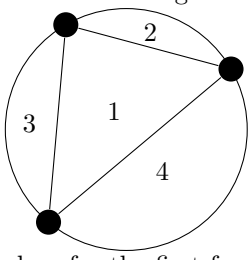
**WORKSHOP 16: §8.1-8.3 MARCH 9, 2017**

Name: _____

(1) Explain why the following are true. (No formal proofs, just good explanations.)
   (a) $\sum_{i=1}^{n} 1 = n$
   (b) $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$
   (c) $\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$
   (d) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. *(Hint: Write the sum in expanded form twice, once in increasing order and again in decreasing order, lining up terms. If you add the two expressions, what do you get?)*
   (e) $\sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r}$. *(Hint: write the sum and $r$ times the sum in expanded form, lining up like terms. If you subtract the two expressions, what do you get?)*

(2) Consider a disk with $n$ points on its boundary circle, and draw all possible line segments joining these points to each other. Let $r_n$ denote the maximum possible number of regions that these segments divide the disk into. (To maximize the number of regions is equivalent to requiring that no three of the segments pass through the same interior point.)




For example, the figure to the left shows that $r_3 = 4$.

   (a) Find $r_n$ for the first few values of $n$.
   (b) Can you guess the closed formula for $r_n$? How would you convince someone (or yourself) that your formula is correct? If you have a convincing argument, convince your neighbors, then Ben.
   (c) If you cannot find a closed formula, find a recurrence relation.

(3) Here are three phrases that sound like they ought to be the same:
   - The sequence $\{a_n\}$ is increasing.
   - The sequence $\{a_n\}$ is non-decreasing.
   - The sequence $\{a_n\}$ is not decreasing.

   But alas, these all mean different things. Find a sequence that is:
   (a) non-decreasing but not increasing
   (b) not decreasing but not increasing
   (c) not decreasing but not non-decreasing

   Actually, the three properties are strengthenings of each other. Prove that
   (d) Every increasing sequence is also non-decreasing.
   (e) Every non-decreasing sequence (of length at least 2) is not decreasing.