

Name: _____

- (1) Compute the following
 - (a) $123 \operatorname{div} 7$
 - (b) $123 \bmod 7$
 - (c) $-12 \operatorname{div} 7$
 - (d) $-12 \bmod 7$
- (2) Use the Division Algorithm to prove that every integer is either even or odd. *[I said earlier that we would have to wait for induction to prove this fact. That's sort of right, because the proof of the Division Algorithm relies on induction.]*
- (3) Use the Division Algorithm and cases to prove: for every odd integer n , $n^2 \equiv 1 \pmod{8}$. *[Hint: consider $n \bmod 4$.]*
- (4) In \mathbb{R} and \mathbb{Z} , we have the *zero product rule*: $xy = 0$ if and only if $x = 0$ or $y = 0$. Does the zero product rule hold in \mathbb{Z}_2 ? \mathbb{Z}_3 ? \mathbb{Z}_4 ? \mathbb{Z}_5 ? \mathbb{Z}_6 ? Conjecture an answer to the question: for which n does \mathbb{Z}_n have the zero product rule?