

Name: \_\_\_\_\_

- (1) For which of the following sets is 2 an element? For which is  $\{2\}$  an element? What is the cardinality of each set?
  - (a)  $\{1, 2, 3\}$
  - (b)  $\{2\}$
  - (c)  $\emptyset$
  - (d)  $\{\emptyset\}$
  - (e)  $\{\{2\}\}$
  - (f)  $\{\{2\}, \{\{2\}\}\}$
  - (g)  $\{2, \{\{2\}\}, \{2, \{2\}\}\}$
- (2) Write the following sets in roster notation:
  - (a)  $\{1, 2\} \times \{1, 3\}$
  - (b)  $\mathcal{P}(\{1, 2\})$
  - (c)  $\mathcal{P}(\{1, \{1\}\})$
- (3) What is the cardinality of  $A \times B$ , in terms of  $|A|$  and  $|B|$ ?
- (4) Prove that if  $\mathcal{P}(A) = \mathcal{P}(B)$ , then  $A = B$ .
- (5) Let  $S = \{1, 2, 3\}$ , and let  $S_i$  be the set of all subsets of  $S$  with cardinality  $i$ . Is  $\{S_0, S_1, S_2, S_3\}$  a partition of  $\mathcal{P}(S)$ ?
- (6)
  - (a) Let  $S = \{1, 2, 3\}$ ,  $X = \mathcal{P}(S)$ , and  $Y$  be the set of all binary strings of length 3. What is the cardinality of  $X$ ? Of  $Y$ ?
  - (b) Pair off the elements of  $X$  and  $Y$  in a meaningful way.
  - (c) Generalize this to  $S = \{1, 2, \dots, n\}$  and  $Y$  the set of binary strings of length  $n$ .
  - (d) Use problem 3 to prove Theorem 3.6.1: for every finite set  $A$ ,  $|\mathcal{P}(A)| = 2^{|A|}$ .