WORKSHOP 8: §3.1-3.4 FEBRUARY 7, 2017

Name: ______________________________

(1) If $A = \{1, 2, 3, 4\}$, $B = \{x : x \text{ is prime}\}$, $C = \{1, 2, 5, 6\}$, find
   (a) $A - B$
   (b) $(A \oplus C) \cap B$
   (c) $B - (A \cup C)$
   (d) $(B - A) \cup (B - C)$ *[Is this the same as the last one? Does $-$ (always) distribute across $\cup$?]*

(2) The predicate "$A \subseteq B$" is defined as $\forall x \; (x \in A \to x \in B)$ (where the domain of discourse is whatever universal set happens to be implied). Write the negation, i.e. the definition of "$A \not\subseteq B$." Interpret the result in plain English.

(3) Use the previous problem to prove that for every set B, $\varnothing \subseteq B$.

(4) *Set builder notation.* Many sets are defined using the notation $\{x : P(x)\}$ for predicate P. This is sometimes called the *truth set* of P, because it consists of all the objects x that make the predicate P true. In this problem, we will denote this set by $TS(P)$. When P can be expressed in terms of smaller predicates, $TS(P)$ can be built with corresponding set operations.

   For example, $TS(P \vee Q) = \{x : P(x) \vee Q(x)\} = \{x : P(x)\} \cup \{x : Q(x)\} = TS(P) \cup TS(Q)$.

   Rewrite each of the following in terms of the truth sets of P and Q.
   (a) $TS(P \wedge Q)$
   (b) $TS(\neg P)$
   (c) $TS(P \to Q)$
   (d) $TS(P \oplus Q)$
   (e) $TS(P \leftrightarrow Q)$

The most common way to prove that two sets A and B are equal is to show that $A \subseteq B$ and $B \subseteq A$. *[Why is this equivalent to the text's definition, $\forall x \; (x \in A \leftrightarrow x \in B)$?]*

(5) Prove the distributive law $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(6) Prove the domination law $A \cap \varnothing = \varnothing$.

(7) Prove that the symmetric difference is associative: $A \oplus (B \oplus C) = (A \oplus B) \oplus C$. (At some point in the proof, use cases.) Use your proof to describe the elements of this set.