WORKSHOP 24: §13.1-3, 13.6 APRIL 20, 2017

Name: ______________________________

A few things you'll need handy from the reading:
the *degree* of a vertex; the Total Degree Formula (Thm13.1.1); Euler's Identity (Thm13.6.1)

(1) We can now provide a closed-form solution to the problem of n points on a circle dividing the disk into regions.

The problem: place n points along the boundary of a disk, and join every pair of them with line segments; what is the maximum number of regions r_n that these segments cut the disk into?

We mentioned that to maximize the number of regions, we should place the points so that no three of the segments meet at a single interior point. We found the first few values:

n	1	2	3	4	5	6	7
r_n	1	2	4	8	16	31	57

And we hinted at how one could define a recurrence relation for $\{r_n\}$, though it wasn't pretty. (All the above was from Workshop 20.)

(a) From the geometric problem, we will construct a (multi)graph: the vertices are the original points together with a vertex at each intersection of two line segments; an edge in this graph is either one of the circular arcs between points or one of the now-smaller line segments between two vertices. Draw in all the vertices on the example below.

(b) The first hard part is to determine how many vertices are in the interior of the disk. Well, any 4-set of points on the circle give rise to a quadrilateral and its two diagonals; these two diagonals intersect at one interior vertex. *E.g., the four colored points and the corresponding segments below.* And conversely, any interior intersection came from two line segments with 4 distinct points on the circle. So the number of interior vertices is the same as the number of 4-sets of points on the circle. [Make sure you believe that first.] How many of those are there?

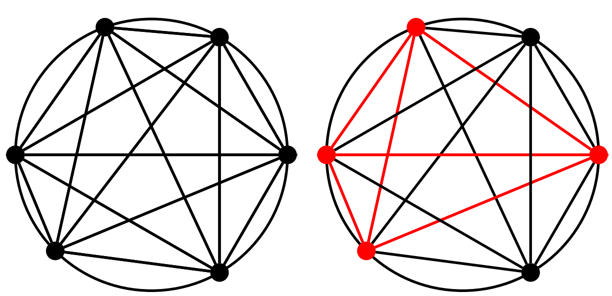
(c) Now we need to count the edges. To do this, we will use the degree-sum formula, so we need to find the degree of every vertex in our graph.

(i) What is the degree of an interior vertex? What's the sum of all these degrees?

(ii) What is the degree of a vertex on the circle? What's the sum of all these degrees?

(iii) So what's the total degree of our graph? Then how many edges are there?

(d) Finally, use Euler's identity: $v - e + f = 2$ where v is the number of vertices, e the number of edges, f the number of faces/regions of a planar embedding of a graph. *[zyBooks uses different names for these objects, but they conflict with the notation for this problem.]* How many faces does this planar embedding have? So what is r_n? Check your formula against the example values above.



(2) Which of the following graphs are equal?
Which are isomorphic? Provide an isomorphism or a reason why they are not isomorphic.