WORKSHOP 23: §10.7, 10.11; CHAPTER 9 APRIL 11, 2017

Name: ____________________

(1) You decide to read one book to your child each night of the week. You have 100 distinct titles.
  (a) With no restrictions, how many weekly lineups are possible?
  (b) If you insist that you read different stories every night of a week, how many lineups are possible?
  (c) If your child insists that you read "Goodnight Loon" exactly once in the week, how many lineups are possible? (Any other book may be repeated.)
  (d) If your child insists that you read "Goodnight Loon" at least once in the week, how many lineups are possible? (Any other book may also be repeated.)

(2) Using Inclusion-Exclusion and Complement-counting, we *can* find the number of surjective functions if the target is small. Consider a domain X with cardinality n. *(The desired answers to the below questions are in terms of n, but you can partially check your work by plugging in small values of n and finding all the requested functions.)*
  (a) How many functions are there from X to $\{a\}$? How many of them are surjective?
  (b) How many functions are there from X to $\{a, b\}$? How many of them are surjective?
  (c) How many functions are there from X to $\{a, b, c\}$? How many of them are surjective?

(3) Prove: for any finite poset $(P, \preceq)$, for any $x \in P$, there exists $y \in P$ such that $x \preceq y$ and y is a maximal element. *(Direct proof, consider $Z = \{z \in P : z \succeq x\}$, apply Workshop20 #1,2.)*

(4) Let $f : X \to Y$ be any function. Define a relation R on X by $x_1 R x_2$ if (and only if) $f(x_1) = f(x_2)$.
  (a) For example, let $X = \{1, 2, 3, 4, 5, 6\}$, $Y = \{a, b, c\}$, $f(1) = f(2) = f(5) = a$, $f(3) = b$, $f(4) = f(6) = c$. Draw the arrow diagram of R.
  (b) Back to generic X, Y, f. Prove that R is an equivalence relation.
  (c) Describe the equivalence classes of f.
  (d) For each $y \in Y$, define $f^{-1}(y) = \{x \in X : f(x) = y\}$. Then the equivalence classes of R are *almost, but not quite* the same as the collection of sets $\{f^{-1}(y)\}_{y \in Y}$. What is the difference?