WORKSHOP 22: §10.5, 10.6, 11.2 APRIL 6, 2017

Name: ____________________

(1) You now have the tools to properly complete questions $3'$(b,c) from last time:

You have 96 distinct childrens' book titles, but one of them ("Goodnight Loon") you have five copies of (for a total of 100 books).

  (c) If you stack all the books, how many different orderings are possible?
  (b) If you choose 40 books to put on a bookshelf, how many different orderings are possible?

(2) You like playing chess, but you always lose against your roommate because they know opening theory and you do not. You suggest arranging the pieces in the first row randomly. [1]

  (a) How many different starting positions are there if your opponent's starting position is the mirror image of your own?
  (b) How many different starting positions are there if your opponent's starting position is determined separately from yours?
  (c) How many different starting positions are there if you must start your bishops on opposite-color squares (and, say, your opponent has the mirror image of your setup)?
  (d) What if you insist that the king is placed with one rook to the left and one rook to the right (not necessarily adjacent to the king)?
  (e) What if you insist on both restrictions from 2c and 2d?

(3) Give a combinatorial proof of the identity $\sum_{k=1}^{n}\left[\binom{n}{k}\cdot k\right] = n2^{n-1}$. *I.e., find a set S and two counting arguments to find $|S|$: one that gives rise to the left side of the identity, and the other the right side.*

(4) Use algebra to show that $k\binom{n}{k} = (n-k)\binom{n}{k-1}$. Then give a combinatorial proof.

[1] One king, one queen, two indistinguishable rooks, two indistinguishable bishops, and two indistinguishable knights are to be placed on eight (distinguishable) squares, which alternate in color white/black.