WORKSHOP 3: §1.9-10 JANUARY 19, 2017

Name: ______________________________

(1) Using the domain $\mathbb{Z}^+$, translate the following into simple English, and determine which are true.
 (a) $\exists x\ \exists y\ x \le y$
 (b) $\exists y\ \exists x\ x \le y$
 (c) $\exists x\ \forall y\ x \le y$
 (d) $\exists y\ \forall x\ x \le y$
 (e) $\forall x\ \exists y\ x \le y$
 (f) $\forall y\ \exists x\ x \le y$
 (g) $\forall x\ \forall y\ x \le y$
 (h) $\forall y\ \forall x\ x \le y$

(2) Which of the following are true (for every domain and every predicate P)?
 (a) $\exists x\ \exists y\ P(x,y) \equiv \exists y\ \exists x\ P(x,y)$
 (b) $\forall x\ \forall y\ P(x,y) \equiv \forall y\ \forall x\ P(x,y)$
 (c) $\forall x\ \exists y\ P(x,y) \equiv \exists y\ \forall x\ P(x,y)$

(3) Why is it true that $\exists x\ \exists y\ (P(x) \wedge P(y)) \equiv \exists z\ P(z)$ (for every domain and every predicate P)?

(4) Consider the domain of all humans and the predicates $S(x,y)$: "x is a sibling of y," $P(x,y)$: "x is a parent of y." *[Please note that you are not asked about the truth value of any of the following.]* Express the following in simple English.
 (a) $\forall x\ \exists y\ P(x,y)$
 (b) $\forall y\ \exists x\ P(x,y)$
 (c) $\forall x\ \exists y\ \exists z\ ((y \ne z) \wedge P(y,x) \wedge P(z,x))$
 (d) $\exists z\ \exists w\ (P(z,x) \wedge P(w,y) \wedge S(z,w))$

 Express the following in formal logic.
 (e) "a has a grandparent"
 (f) "b has at least three siblings"
 (g) "c has exactly one sibling."
 (h) "Being siblings is a reciprocal relationship."
 (i) "Parenthood is a never-reciprocal relationship."

(5) Express "whenever two numbers are distinct, there is a number strictly between them" in formal logic. (The domain is $\mathbb{R}$.)

(6) In calculus, the definition of "$\lim_{x\to a} f(x) = L$" is

$$\forall \varepsilon\ \exists \delta\ \forall x\ (0 < |x-a| < \delta \to |f(x) - L| < \varepsilon).$$

Simplify the negation of this predicate as much as possible (there should be no $\neg$ symbol in the end).

(7) Some people use the symbols $\exists!$ to denote "there exists a unique...". Find a logically equivalent proposition to $\exists! x\ P(x)$ that does not use this new quantifier (just the ordinary $\exists$ and propositional operators).