WORKSHOP 2: §1.6-8 JANUARY 17, 2017

Name: ____

In mathematical English, "for any" sometimes means existential and sometimes universal quantification. Please avoid using "for any" in this class!

(1) Which of the following are predicates, and which are propositions? For predicates, how many free variables are there? For propositions, are they true or false?
  (a) $x + y - z = 2$
  (b) $\exists x\ (x + y - 5 = 2)$
  (c) There is an integer x whose square is y.
  (d) There is an integer x whose square is 5.
  (e) Every real number has a nonnegative square.

For a quantified implication, the converse, inverse, and contrapositive are defined as the quantified version of the same. (I.e., the converse of "$\forall x(P(x) \to Q(x))$" is "$\forall x(Q(x) \to P(x))$.")

In English, sometimes universal quantification is implicit. "If a day is Thanksgiving, then the next day is Friday" probably is meant as the proposition "for each day, if that day is Thanksgiving, then the next day is Friday" (and not as the predicate whose truth value depends on the day in question).

(2) Consider the proposition "if a day is Thanksgiving, then the next day is Friday."
  (a) Is it true or false?
  (b) What is its converse? Is that true or false?
  (c) What is its inverse? Is that true or false?
  (d) What is its contrapositive? Is that true or false?
  (e) What is its negation? Is that true or false?

(3) For each of the following, write the proposition in formal logic, then write its negation (also in formal logic), then write the negation in natural language. (Don't just say "it is not the case that ... .")
  (a) Every rational number's square is also a rational number.
  (b) There is a real number whose square is itself.

(4) *Restricting/Expanding Domains.* Rewrite each of the following as an equivalent proposition using the domain $\mathbb{R}$ instead of $\mathbb{R}^+$ (which is the set of positive real numbers). *(Hint: the predicate being quantified will need something added to it to account for the different domain.)*
  (a) $\exists x \in \mathbb{R}^+\ (x^2 = 1)$
  (b) $\forall x \in \mathbb{R}^+\ (x^2 = 1)$

(5) Do quantifiers distribute over operations? Specifically:
  (a) Is it true that $\exists x(P(x) \lor Q(x)) \equiv (\exists x\ P(x)) \lor (\exists x\ Q(x))$?
  (b) Is it true that $\exists x(P(x) \land Q(x)) \equiv (\exists x\ P(x)) \land (\exists x\ Q(x))$?
  (c) Is it true that $\forall x(P(x) \lor Q(x)) \equiv (\forall x\ P(x)) \lor (\forall x\ Q(x))$?
  (d) Is it true that $\forall x(P(x) \land Q(x)) \equiv (\forall x\ P(x)) \land (\forall x\ Q(x))$?

  As always, you should justify your answer. Here, if you answer "no," you should provide a counterexample, i.e. predicates P and Q and a domain of discourse for which the two propositions have different truth values; if you answer "yes," then you should explain why the two propositions are the same no matter what P, Q, and domain are being used.

(6) For this problem, the domain of discourse is $\mathbb{Z}$, the set of integers. Consider the predicates Odd(x): "x is odd," Prime(x): "x is prime," and Square(x): "x is a perfect square." Rewrite each of the following in natural language and determine whether true or false. *Note: 1 is not a prime.*
  (a) $\exists x(\text{Odd}(x) \land \text{Square}(x))$
  (b) $\forall x(\text{Prime}(x) \to \neg\,\text{Square}(x))$
  (c) $\forall x(\text{Prime}(x) \to \text{Odd}(x) \lor x = 2)$

(7) The proposition $\forall x \exists x P(x) \lor Q(x)$ makes sense using our order of operations, but it is not very clear. Rewrite the proposition so that it is clearer (but means the same thing).