WORKSHOP 5: §1.13-2.1 JANUARY 26, 2017

Name: ______________________________

Key concepts:

For an argument with quantifiers, to prove validity you need to use the laws in Table 1.13.1 (together with earlier things). To prove invalidity, you need to produce a situation (domain of discourse and predicates) for which all the hypotheses are true but the conclusion is false. Drawing a Venn diagram may be helpful to decide whether an argument is valid or not.

Remember, when we present an argument in English and ask whether it is valid, we really mean whether the underlying argument form is valid. We do not care about whether any of the propositions involved are actually true or false.

(1) Determine which of the following are valid arguments.

(a) Every girl scout who sells at least 50 boxes of cookies will get a prize. Suzy, a girl scout, got a prize. Therefore Suzy sold at least 50 boxes of cookies.

(b) Every girl scout who sells at least 50 boxes of cookies will get a prize. Suzy, a girl scout, did not get a prize. Therefore Suzy sold less than 50 boxes of cookies.

(c) Every perfect square is not prime. 17 is not a perfect square. Therefore 17 is prime.

(d) Every big city has tall buildings. Chicago does not have tall buildings. Therefore Chicago is not a big city.

(e)
$$\begin{array}{rl} & \exists x\ (P(x) \land Q(x)) \\ \hline \therefore & \exists x\ P(x) \land \exists x\ Q(x) \end{array}$$

(f)
$$\begin{array}{rl} & \exists x\ P(x) \land \exists x\ Q(x) \\ \hline \therefore & \exists x\ (P(x) \land Q(x)) \end{array}$$

(g)
$$\begin{array}{rl} & \forall x\ (P(x) \to Q(x)) \\ & \forall y\ (\neg Q(y) \lor R(y)) \\ & \exists z\ \neg P(z) \\ \hline \therefore & \exists w\ R(w) \end{array}$$

(h)
$$\begin{array}{rl} & \forall x\ (P(x) \to Q(x)) \\ & \forall y\ (Q(y) \to R(y)) \\ \hline \therefore & \forall z\ (P(z) \to R(z)) \end{array}$$

(i)
$$\begin{array}{rl} & \exists x\ (P(x) \to Q(x)) \\ & \exists y\ (Q(y) \to R(y)) \\ \hline \therefore & \exists z\ (P(z) \to R(z)) \end{array}$$

(2) Prove or disprove the following.

(a) Every prime number is odd.

(b) The first four *Mersenne numbers* – numbers of the form $2^p - 1$, where p is a prime – are prime.

(c) All Mersenne numbers are prime.

(d) Every even integer between 4 and 26 can be written as the sum of two primes.

(3) Complete the following proof that the square of any odd integer is odd.

Let n be an arbitrary odd integer.
By the definition of odd, $n =$ for some integer k.
Then $n^2 =$.
Since k is an integer, $2k^2 + 2k$ is an integer *[Why?]*.
Thus n^2 is odd.

(4) Find the flaw in the following "proof" that the average of two odd integers is even.

Let m and n be arbitrary odd integers.
The average of m and n is $\dfrac{m+n}{2} = 2\left(\dfrac{m+n}{4}\right)$.
Since m and n are integers, $\frac{m+n}{4}$ is an integer.
Thus the average is twice an integer, i.e. even.