WORKSHOP 4: §1.11-13   JANUARY 24, 2017

Name: ______________________________

Key concepts:

An argument consists of some number of *hypotheses* $p_1, \dots, p_n$ and a *conclusion* q.

An argument is *valid* if whenever the hypotheses are true, so is the conclusion. It is *invalid* otherwise, i.e. if there is some situation when the hypotheses are all true but the conclusion is not. In other words, validity is the same as $(p_1 \wedge \cdots \wedge p_n) \to q$ being a tautology. If all the propositions depend on some propositional variables and there are no quantifiers involved, then this can be checked as before using a truth table, OR using a slight shortcut: validity means that for every row in the truth table where all the hypotheses are true, so must also be the conclusion (and invalidity means there is some row where all the hypotheses are true but the conclusion is false).

As with logical equivalence, it is convenient to have some basic valid arguments and use those to build up more complicated ones (rather than always creating a truth table, which sometimes may be quite large). See Tables 1.12.1 and 1.13.1.

(1) Verify that the argument Resolution is valid:

$$\begin{array}{ll} & p \vee q \\ & \neg p \vee r \\ \hline \therefore & q \vee r \end{array}$$

(2) Prove that the following argument is valid using Table 1.12.1. *(Hint: rewrite implications using logical equivalences first.) [This argument is the logical basis for Proofs by Cases later.]*

$$\begin{array}{lll} & s \to u & (a) \\ & t \to u & (b) \\ & s \vee t & (c) \\ \hline \therefore & u & \end{array}$$

(3) Prove that the following argument is valid using Table 1.12.1 and (if you find it convenient) the result of question (2).

$$\begin{array}{lll} & s \to t & (a) \\ & \neg t \vee u & (b) \\ & t \vee s & (c) \\ \hline \therefore & u & \end{array}$$

(4) A very common **invalid** argument is $p \to q$, $\neg p$, $\therefore \neg q$. Explain why it is invalid. Take care not to use this!

(5) Determine which of the following are valid arguments.

(a) Every girl scout who sells at least 50 boxes of cookies will get a prize. Suzy, a girl scout, got a prize. Therefore Suzy sold at least 50 boxes of cookies.

(b) Every girl scout who sells at least 50 boxes of cookies will get a prize. Suzy, a girl scout, did not get a prize. Therefore Suzy sold less than 50 boxes of cookies.

(c) Every perfect square is not prime. 17 is not a perfect square. Therefore 17 is prime.

(d) Every big city has tall buildings. Chicago does not have tall buildings. Therefore Chicago is not a big city.

(e)
$$\begin{array}{ll} & \exists x\ (P(x) \wedge Q(x)) \\ \hline \therefore & \exists x\ P(x) \wedge \exists x\ Q(x) \end{array}$$

(f)
$$\begin{array}{ll} & \exists x\ P(x) \wedge \exists x\ Q(x) \\ \hline \therefore & \exists x\ (P(x) \wedge Q(x)) \end{array}$$

(g)
$$\begin{array}{ll} & \forall x\ (P(x) \to Q(x)) \\ & \forall y\ (\neg Q(y) \vee R(y)) \\ & \exists z\ \neg P(z) \\ \hline \therefore & \exists w\ R(w) \end{array}$$

(h)
$$\begin{array}{ll} & \exists x\ \forall y\ P(x,y) \\ \hline \therefore & \forall y\ \exists x\ P(x,y) \end{array}$$