

QUIZ 4: CHAPTER 6 APRIL 4

Name: _____

- All answers should be fully justified.
- Complete this quiz without any aids, including the text or your peers.

(1) Consider the following mystery code.

```
Input:   $n$ , a positive integer
        a sequence of nonnegative integers,  $L = (a_1, a_2, \dots, a_n)$ , all at most  $5^n$ 
        (i.e.,  $0 \leq a_i \leq 5^n$  for all  $i$ )
Output: a number...???
```



```
 $L := \text{sort}( L )$ 
While (  $a_2 \neq 0$  )
  For  $i = 1$  to  $n - 1$ 
     $a_i := a_i - a_{i+1}$ 
  End-for
   $L := \text{sort}( L )$ 
End-while
```



```
Return(  $a_1$  )
```

The function `sort` called has asymptotic time complexity $\Theta(n \log n)$ and returns the input sequence sorted in nonincreasing order.

(a) Run the algorithm on the input $n = 3$, $L = (10, 8, 2)$. Write down the value of L after each iteration of the `While` loop.

(b) It is true that the `While` loop is executed at most $n \cdot 5^n$ many times. Use this to give an asymptotic upper bound on the time complexity for this algorithm.

(c) *Bonus:* What does the algorithm do? (Make sure to complete the rest of the quiz before trying this.)

- (2) Let $f(n) = n^5 - 17n^4 + 3n + 7$.
- (a) Prove formally that $f(n) = O(n^5)$. (Use only the definition of $O(\cdot)$, no theorems about $O(\cdot)$ notation are allowed.)

- (b) Prove formally that $f(n) = \Omega(n^5)$. (Again, use only the definition of $\Omega(\cdot)$.)