QUIZ 4: CHAPTER 6 APRIL 4

Name: ______________________________

- All answers should be fully justified.
- Complete this quiz without any aids, including the text or your peers.

(1) Consider the following mystery code.

```
Input:   n, a positive integer
         a sequence of nonnegative integers, L = (a_1, a_2, ..., a_n), all at most 5^n
            (i.e., 0 ≤ a_i ≤ 5^n for all i)
Output: a number...???

L := sort( L )
While ( a_2 ≠ 0 )
   For i = 1 to n − 1
      a_i := a_i − a_{i+1}
   End-for
   L := sort( L )
End-while

Return( a_1 )
```

The function sort called has asymptotic time complexity $\Theta(n \log n)$ and returns the input sequence sorted in nonincreasing order.

(a) Run the algorithm on the input $n = 3$, $L = (10, 8, 2)$. Write down the value of L after each iteration of the While loop.

$(10, 8, 2)$
in For, $i=1$: $(2, 8, 2)$
$i=2$: $(2, 6, 2)$
Sort $(6, 2, 2)$

$(6, 2, 2)$
$(4, 2, 2)$
$(4, 0, 2)$
$(4, 2, 0)$

$(4, 2, 0)$
$(2, 2, 0)$
$(2, 2, 0)$
$(2, 2, 0)$

$(2, 2, 0)$
$(0, 2, 0)$
$(0, 2, 0)$
$(2, 0, 0)$
End-while

output $= 2$

(b) It is true that the While loop is executed at most $n \cdot 5^n$ many times. Use this to give an asymptotic upper bound on the time complexity for this algorithm.

$$\text{time} \lesssim \underbrace{n \log n}_{\text{sort}} + \underbrace{n5^n}_{\text{While}} \cdot \underbrace{\left(\underbrace{(n-1)}_{\text{For}} \cdot \underbrace{(1)}_{\text{inside For}} + n \log n \right)}_{\text{inside While}} = O(n^2 5^n \log n)$$

(c) *Bonus:* What does the algorithm do? (Make sure to complete the rest of the quiz before trying this.)

Finds the gcd of the list

(2) Let $f(n) = n^5 - 17n^4 + 3n + 7$.

(a) Prove formally that $f(n) = O(n^5)$. (Use only the definition of $O(\cdot)$, no theorems about $O(\cdot)$ notation are allowed.)

$$n^5 - 17n^4 + 3n + 7 \leq n^5 + 0 + 3n^5 + 7n^5$$
$$= 11n^5$$

so choosing $n_0 = 1$ & $C = 11$, $f(n) \leq Cn^5$,

i.e. $f(n) = O(n^5)$.

(b) Prove formally that $f(n) = \Omega(n^5)$. (Again, use only the definition of $\Omega(\cdot)$.)

$$n^5 - 17n^4 + 3n + 7 \geq n^5 - 17n^4 + 0 + 0$$
$$= n^4(n-17)$$
$$\geq n^4 \cdot \tfrac{1}{2}n \qquad \underline{\text{if}} \quad n - 17 \geq \tfrac{1}{2}n$$
$$= \tfrac{1}{2}n^5 \qquad \Longleftrightarrow \quad n \geq 34$$

so choosing $n_0 = 34$, $C = \frac{1}{2}$,

whenever $n \geq 34$, $f(n) \geq \frac{1}{2}n^5$,

i.e. $f(n) = \Omega(n^5)$.