

QUIZ 4: CHAPTER 6 APRIL 4

Name: _____

- All answers should be fully justified.
- Complete this quiz without any aids, including the text or your peers.

(1) Consider the following mystery code.

```

Input:  n, a positive integer
        a sequence of nonnegative integers,  $L = (a_1, a_2, \dots, a_n)$ , all at most  $5^n$ 
        (i.e.,  $0 \leq a_i \leq 5^n$  for all  $i$ )
Output: a number...???

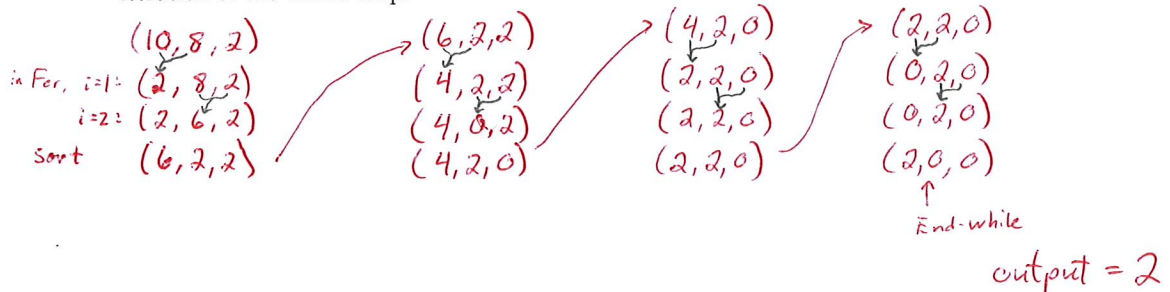
L := sort( L )
While (  $a_2 \neq 0$  )
  For  $i = 1$  to  $n - 1$ 
     $a_i := a_i - a_{i+1}$ 
  End-for
  L := sort( L )
End-while

Return(  $a_1$  )

```

The function `sort` called has asymptotic time complexity $\Theta(n \log n)$ and returns the input sequence sorted in nonincreasing order.

(a) Run the algorithm on the input $n = 3$, $L = (10, 8, 2)$. Write down the value of L after each iteration of the While loop.



(b) It is true that the While loop is executed at most $n \cdot 5^n$ many times. Use this to give an asymptotic upper bound on the time complexity for this algorithm.

$$\text{time} \lesssim \underbrace{n \log n}_{\text{sort}} + \underbrace{n 5^n \cdot \left((n-1) \cdot \underbrace{\left(\frac{1}{5} \right)}_{\substack{\text{inside} \\ \text{For}}} + n \log n \right)}_{\text{inside While}} = O(n^2 5^n \log n)$$

(c) Bonus: What does the algorithm do? (Make sure to complete the rest of the quiz before trying this.)

Finds the gcd of the list

(2) Let $f(n) = n^5 - 17n^4 + 3n + 7$.

(a) Prove formally that $f(n) = O(n^5)$. (Use only the definition of $O(\cdot)$, no theorems about $O(\cdot)$ notation are allowed.)

$$\begin{aligned} n^5 - 17n^4 + 3n + 7 &\leq n^5 + 0 + 3n^5 + 7n^5 \\ &= 11n^5 \end{aligned}$$

so choosing $n_0 = 1$ & $C = 11$, $f(n) \leq Cn^5$,

i.e. $f(n) = O(n^5)$.

(b) Prove formally that $f(n) = \Omega(n^5)$. (Again, use only the definition of $\Omega(\cdot)$.)

$$\begin{aligned} n^5 - 17n^4 + 3n + 7 &\geq n^5 - 17n^4 + 0 + 0 \\ &= n^4(n - 17) \\ &\geq n^4 \cdot \frac{1}{2}n \quad \text{if } n - 17 \geq \frac{1}{2}n \\ &= \frac{1}{2}n^5 \quad \Leftrightarrow n \geq 34 \end{aligned}$$

so choosing $n_0 = 34$, $C = \frac{1}{2}$,

whenever $n \geq 34$, $f(n) \geq \frac{1}{2}n^5$,

i.e. $f(n) = \Omega(n^5)$.