QUIZ 3: CHAPTER 4 MARCH 28

Name: _____

- All answers should be fully justified.
- Complete this quiz without any aids, including the text or your peers.

(1) True or false: For every $x \in \mathbb{Q}$, $2\lfloor x \rfloor = \lfloor 2x \rfloor$. Prove your answer.

Let $x = 0.5$. Then $2\lfloor x \rfloor = 2 \cdot 0 = 0$

but $\lfloor 2x \rfloor = \lfloor 1 \rfloor = 1$.

(2) True or false: If $f : A \to B$ and $g : B \to C$ are both surjective, then $g \circ f$ is surjective. Prove your answer.

Direct proof. Let $f: A \to B$ & $g: B \to C$ be (arbitrary) surjective functions.

Let $z \in C$. Since g is surjective, $\exists y \in B$ s.t. $g(y) = z$.

Since f is surjective, $\exists x \in A$ s.t. $f(x) = y$.

Then $g \circ f(x) = g(f(x)) = g(y) = z$.

So $g \circ f$ is surjective. □

(3) Prove that for every $n \in \mathbb{Z}$, $\left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil = n$.

Proof by cases.

Case 1: n is even. Let $n = 2k$, $k \in \mathbb{Z}$. Then $\left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil = \left\lfloor \frac{2k}{2} \right\rfloor + \left\lceil \frac{2k}{2} \right\rceil$

$= \lfloor k \rfloor + \lceil k \rceil$

$= k + k$ (since $k \in \mathbb{Z}$)

$= 2k$

$= n$.

Case 2: n is odd. Let $n = 2k+1$, $k \in \mathbb{Z}$. Then

$\left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil = \left\lfloor \frac{2k+1}{2} \right\rfloor + \left\lceil \frac{2k+1}{2} \right\rceil$

$= \left\lfloor k + \frac{1}{2} \right\rfloor + \left\lceil k + \frac{1}{2} \right\rceil$

$= k + \left\lfloor \frac{1}{2} \right\rfloor + k + \left\lceil \frac{1}{2} \right\rceil$ ($\lfloor \cdot \rfloor$ & $\lceil \cdot \rceil$ behave well on integer addition, $k \in \mathbb{Z}$)

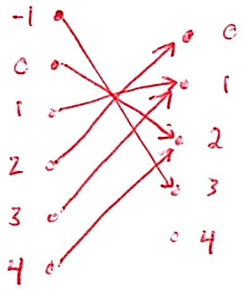
$= k + 0 + k + 1$

$= 2k + 1$

$= n$. □

(4) Define $f : \{-1, 0, 1, 2, 3, 4\} \to \{0, 1, 2, 3, 4\}$ by $f(x) = |x - 2|$.

(a) Draw an arrow diagram for f.



(b) Give the set representation of f.

$f = \{(-1, 3), (0, 2), (1, 1), (2, 0), (3, 1), (4, 2)\}$

(c) Find the range of f.

$\{0, 1, 2, 3\}$

(d) Is f onto? (Why or why not?)

No, 4 is in the target but not the range.

(e) Is f one-to-one? (Why or why not?)

No, 1 & 3 both have 1 as their output.