

Name: \_\_\_\_\_

- Please refer to the syllabus regarding allowed collaboration on this homework assignment.
- All answers should be fully justified.
- Your homework should be neatly written on additional paper; you may attach this cover page if you would like to keep the questions attached to the answers.

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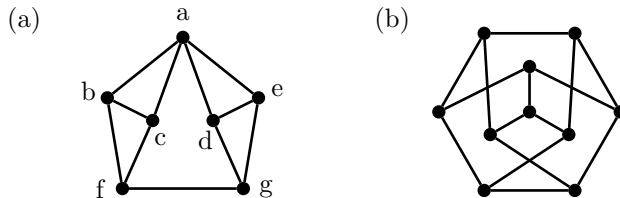
- (1) A university has only three majors in the sciences: there are 124 computer science students, 83 math students, and 160 physics students. Many students dual-major, and these are counted in both of the relevant categories above; there are 30 CS/math students, 10 math/physics students, and 22 physics/CS students. Four students are triple-majoring (CS/math/physics).

How many students are *not* math majors?

- (2) For each part, either find a simple graph with the properties or prove that none exists. (*Experiment. You may find useful the Total Degree Formula, Theorem 13.6.2, and/or Theorem 14.3.3.*)
- has vertex degrees 6,3,2,1,1,1
  - has vertex degrees 5,3,2,1,1,1
  - has vertex degrees 4,3,2,1,1,1
  - is planar, has 7 vertices and 14 edges
  - is planar, has 7 vertices and 21 edges

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- is a tree, has 10 vertices and 11 edges
  - is a tree, has a vertex of degree 7 and exactly 10 leaves
- (3) Let  $G$  be a graph with  $n$  vertices and  $m$  edges. If  $G$  is planar, then  $m \leq 3n - 6$ . Prove that the converse is false, by constructing a graph with  $n$  vertices and at most  $3n - 6$  edges that is not planar. (*See Exercise 13.6.5*)
- (4) Here are two (moderately famous) graphs. What are their chromatic numbers? Prove your answers. (*You may find Exercise 13.7.2(c) useful.*)



- (5) *Prefix trees*
- Why do prefix trees only have code letters at leaves? As a specific example, why does the first tree below not give a prefix code?
  - Why, in a prefix tree, does every non-leaf have two children (never just one)? As a specific example, why does the second tree give rise to a poor prefix code? (*The resulting code works, it's just not as efficient as it could be.*)
  - Decode 11010001000101 using the third tree below.

