HW 12

1) #non-math = total # students − # math students

$$= |C \cup M \cup P| - |M|$$

$$= \left(|C| + \cancel{|M|} + |P| - |C \cap M| - |C \cap P| - |M \cap P| + |C \cap M \cap P| \right) - \cancel{|M|}$$

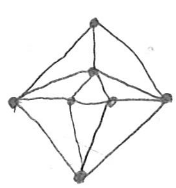
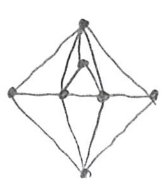
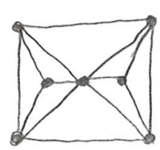
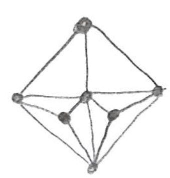
$$= 124 + 160 - 30 - 22 - 10 + 4$$

$$= 226.$$

2) a) none; each vertex can have at most all 5 other vertices as neighbors, so there cannot be a vertex of degree 6.

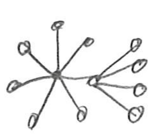
b) none; the total degree is 13, which is odd, but the Total Degree Formula implies that the total degree of any graph is even.

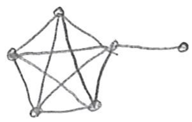
c)  [this is the only such graph, up to isomorphism]

d)  OR  OR  OR  OR . . .

e) none; a planar 7-vertex graph has $\le 3(7) - 6 = 15$ edges

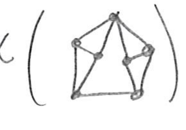
f) none; a 10-vertex tree has $10 - 1 = 9$ edges, not 11

g)  or . . .

3) $G=$  has 6 vertices, 11 edges.

$11 \le 3\cdot 6-6 = 12$ ✓

but G is not planar because it has K_5 as a subgraph.

4) a) $\chi\Big($  $\Big) = 4.$

$\chi \le 4$ since it is planar.

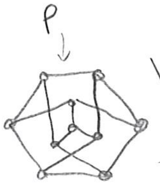
$\chi > 3$: Suppose to the contrary that there is a valid 3-coloring.

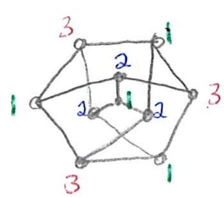
Then since $\{a,b,c\}$ is a clique, these must receive the three colors, say
$a \longleftrightarrow$ yellow
$b \longleftrightarrow$ blue
$c \longleftrightarrow$ cyan .

Since $\{b,c,f\}$ is also a clique, f cannot be blue or cyan, so must be yellow.

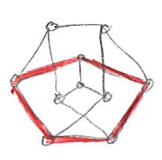
Since $\{a,d,e\}$ is a clique, d & e must be blue & cyan (in either order), and since $\{d,e,g\}$ is too, g must be yellow.

But then the edge fg has yellow on both endpoints, contradicting that the coloring was valid.

b) $\chi\Big($  $\Big) = 3.$

$\chi \le 3$: 

is a valid coloring.

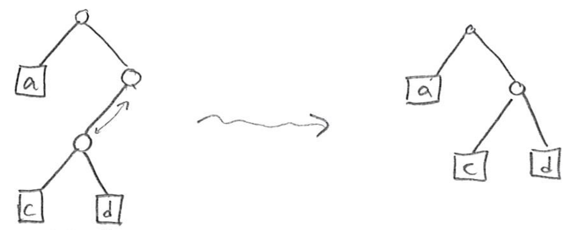
$\chi \ge 3$:  shown is a C_5 as a subgraph,

so $\chi(P) \ge \chi(C_5) = 3$.

5) a) The code for a non-leaf (e.g b) will be a prefix of the code for any descendents (e.g. c, d, e, f)

b) Merging a vertex with just one child with its child reduces the length of the code for each of its leaf descendants (without affecting any other code letters). This is more efficient.

E.g.,



a: 0
c: 100
d: 101

a: 0
c: 10
d: 11

c) queued