HOMEWORK 6: CHAPTER 4 AND §6.1-6.2 DUE FEBRUARY 23

Name: ____

- Please refer to the syllabus regarding allowed collaboration on this homework assignment.
- All answers should be fully justified.
- Your homework should be neatly written on additional paper; you may attach this cover page if you would like to keep the questions attached to the answers.

For the first four problems, let $f : X \to Y$ and $g : Y \to Z$.

(1) Prove that if f and g are both injective, then so is $g \circ f$.

(2) If f is not injective, is it possible that $g \circ f$ is injective? If so, give an example (perhaps with an arrow diagram); if not, prove why it is impossible (and it might help to write down that statement formally).

(3) If g is not injective, is it possible that $g \circ f$ is injective? If so, give an example (perhaps with an arrow diagram); if not, prove why it is impossible (and it might help to write down that statement formally).

(4) The text defines f^{-1} for any function, then goes on to say that f^{-1} is only a function when f is a bijection. Why is that? Specifically: if f is not injective, then what part of the definition of function does f^{-1} violate? If f is not surjective, then what part of the definition of function does f^{-1} violate?

(5) Write an algorithm in pseudocode that takes as input a sequence of real numbers $(a_1, \ldots, a_n)$ and the length n (which is ≥ 2), and outputs the second-smallest number in the list. (If the smallest number is repeated in the list, output that number.)

(6) Let f, g, h be functions from $\mathbb{R}^+$ to $\mathbb{R}^+$. Prove that if $f = O(h)$ and $g = O(h)$, then $f + g = O(h)$. *[This is from Figure 6.2.2; no, you cannot just cite that Figure.]*

(7) Complete the Challenge Activities in §4.2-4.3.

"This is a one line proof...if we have sufficiently wide paper."