HOMEWORK 3: §1.11-2.1 DUE FEBRUARY 2

Name: _____

- Please refer to the syllabus regarding allowed collaboration on this homework assignment.
- All answers should be fully justified.
- Your homework should be neatly written on additional paper; you may attach this cover page if you would like to keep the questions attached to the answers.

(1) Show that the following argument is invalid.

$p \to (q \vee r)$
$\neg r$
$\therefore \quad \neg p$

(2) Use Table 1.12.1 to prove that the following argument is valid. *[If you were to do this with a truth table, it would need 64 rows!]*

$p \vee q$	(i)
$q \to r$	(ii)
$p \wedge s \to t$	(iii)
$\neg r$	(iv)
$\neg q \to u \wedge s$	(v)
$\therefore \quad t$	

(3) Determine whether the following arguments are valid or invalid. *(To prove validity, use Tables 1.12.1 and 1.13.1 (and perhaps 1.5.1). To prove invalidity, produce a domain and predicates that make the hypotheses true and the conclusion false.)*

(a)

$\forall x\ (P(x) \to (Q(x) \wedge R(x)))$	(i)
$\exists x\ P(x)$	(ii)
$\therefore \quad \exists x\ R(x)$	

(b)

$\forall x\ (P(x) \to Q(x))$	(i)
$\exists x\ P(x)$	(ii)
$\therefore \quad \forall x\ Q(x)$	

(4) Determine whether the following argument is valid.

Every cat is striped.
No cat is friendly.
Therefore nothing striped is friendly.

(5) Prove or disprove the following statements.

(a) Every odd positive integer up to 13 is either a square or a prime.
(b) Every integer in $\{-3, -2, -1, 0, 1, 2, 3\}$ is even or odd. *(We have not proven yet, and you may not use here, the fact that every integer is even or odd but not both.)*
(c) For every integer n with $0 \leq n \leq 11$, $n^2 - n + 11$ is prime.

"Divide fourteen sugar cubes into three cups of coffee so that each cup has an odd number of sugar cubes in it."
"That's easy: one, one, and twelve."
"But twelve isn't odd!"
"Twelve is an odd number of cubes to put in a cup of coffee..."