HW3 §1.11 - 2.1

1) p:T q:T r:F make both hypotheses true but the conclusion false

2) (a) $\neg q$ Modus Tollens, (ii), (iv)

(b) p Disj. Syll. (i), (a)

(c) $u \wedge s$ Modus Ponens, (a), (v)

(d) s Simplification, (c)

(e) $p \wedge s$ Conjunction, (b), (d)

(f) t Modus Ponens, (e), (iii)

3) a) Valid

(A) c is particular $\wedge$ P(c) — Exist. Inst., (ii)
(A1) c is particular (A2) P(c) — Simplification, (A)

(B) $P(c) \rightarrow (Q(c) \wedge R(c))$ — Univ. Inst., (i), (A1)

(C) $Q(c) \wedge R(c)$ — Modus Ponens, (B), (A2)

(D) R(c) — Simplification, (C)

(E) $\exists x\ R(x)$ — Exist. Gen., (D)

b) Invalid.

Domain = {a, b}

| | P | Q | P → Q |
|---|---|---|---|
| a | T | T | T |
| b | F | F | T |

$\exists x\ P(x)$ is true (with x=a)

$\forall x\ (P(x) \rightarrow Q(x))$ is true

But $\forall x\ Q(x)$ is false (counterexample: x=b)

OR

(a student answer:)
Domain = $\mathbb{R}$,

$P(x) =$ "$x=1$" $= Q(x)$

$\forall x$, if $x=1$ then $x=1$ ✓

$\exists x : x=1$ ✓

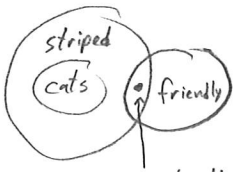
But $\forall x : x=1$ is false (e.g. $x=2$)

4) Invalid: perhaps zebras are friendly

Venn diagram (to help, not as a proof!)

"Every cat is striped":


"No cat is friendly":


Maybe there is a striped friendly non-cat!

5) a) True. Proof by exhaustion:

$1 = 1^2$ $\quad$ $9 = 3^2$

3 is prime $\quad$ 11 is prime

5 is prime $\quad$ 13 is prime

7 is prime

b) True. $-3 = 2(-2)+1$ is odd $\quad$ $-2 = 2(-1)$ is even

$-1 = 2(-1)+1$ - - - $\quad$ $0 = 2(0)$ - - -

$1 = 2(0)+1$ - - - $\quad$ $2 = 2(1)$ - - -

$3 = 2(1)+1$ - - -

c) False. Counterexample $n = 11$: $11^2 - 11 + 11 = 11^2$ is not prime.