Name: _____

- **READ THE FOLLOWING DIRECTIONS!**

- **Do NOT open the exam until instructed to do so.**

- You have seventy-five (75) minutes to complete this exam. When you are told to stop writing, do it or you will lose all points on the page(s) you write on.

- You may not communicate with other students during this test.

- Keep your eyes on your own paper.

- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctor.

- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).

- Any case of cheating will be taken extremely seriously.

- Show all your work and explain your answers when appropriate.

- Before turning in your exam, check to make certain you've answered all the questions.

- The tables of common logical equivalences and valid arguments appear on the back page of the exam.

Question	Points	Score
1	10	
2	12	
3	10	
4	10	
5	6	
6	6	
7	6	
8	8	
9	8	
10	8	
11	6	
12	8	
13	8	
Total:	106	

No justification is necessary on this page.

1. (10 points) Circle "Yes" for propositions and "No" for non-propositions.

   (a) Yes   No   "Is every function computable?"

   (b) Yes   No   "Every function is computable."

   (c) Yes   No   "Accessing the homework requires your Hawk account."

   (d) Yes   No   "Access the homework using your Hawk account."

   (e) Yes   No   "$x^2 = -1$."

2. (12 points) Circle "True" or "False" as appropriate.

   (a) True   False $p \to \neg p$ is a contradiction.

   (b) True   False $(p \leftrightarrow q) \vee (p \oplus q)$ is a tautology.

   (c) True   False $n \mid 0$ is true for every integer n.

   (d) True   False $2 \subseteq \{1, 2, 3\}$.

   (e) True   False $\varnothing \subseteq \{1, 2, 3\}$.

   (f) True   False $\left| \mathcal{P}\big(\mathcal{P}(\varnothing)\big) \right| = 2$.

3. (10 points) The table below records the truth values of $P(x, y)$ where the domain for x is $\{x_1, x_2, x_3, x_4\}$ and the domain for y is $\{y_1, y_2, y_3, y_4\}$. For example, $P(x_2, y_3)$ is true by looking in the 2nd row and 3rd column; similarly, $P(x_2, y_4)$ is true.

Determine whether each of the following is True or False.

   (a) True   False $\exists x \; \exists y \; P(x, y)$

$x \setminus y$	y_1	y_2	y_3	y_4
x_1	F	F	F	T
x_2	T	F	T	T
x_3	F	T	F	T
x_4	T	T	F	T

   (b) True   False $\exists x \; \forall y \; P(x, y)$

   (c) True   False $\forall y \; \exists x \; P(x, y)$

   (d) True   False $\exists y \; \forall x \; P(x, y)$

   (e) True   False $\forall x \; \exists y \; P(x, y)$

   (f) True   False $\forall x \; \forall y \; P(x, y)$

4. (10 points) Circle "Yes" or "No" as appropriate. If the answer is Yes, nothing else is needed. If the answer is No, prove why.

   (a) Yes   No   Consider the set A of binary strings of length 4, and define
                                     $S_1 = \{x \in A : x \text{ starts with } 1\}$, $S_{00} = \{x \in A : x \text{ starts with } 00\}$.
                                     Do S_1, S_{00} form a partition of A?

   (b) Yes   No   Let $B = \{1, 2, 3, 4\}$ and $C = \{3, 4, 5, 6, 7\}$, and let $D = B \times C$.
                                     For each $b \in B$, define $S_b = \{(b, x) : x \in C\}$.
                                     Do S_1, S_2, S_3, S_4 form a partition of D?

   (c) Yes   No   For each $n \in \mathbb{N}$, let $S_n = \{q \in \mathbb{Q} : \exists b \ (b \neq 0 \wedge q = n/b)\}$.
                                     Do $S_0, S_1, S_2, S_3, \ldots$ form a partition of $\mathbb{Q}$?

5. (6 points) Let $p = $ "Alice is smart," $q = $ "Alice studies," and $r = $ "Alice gets an A on the exam." Translate the proposition

   "Alice is smart, but if she doesn't study, she won't get an A on the exam."

   into formal logic, using the propositional variables p, q, r and any connectives.

6. (6 points) Let $P(x) = $ "x is smart," $Q(x) = $ "x studies," and $F(x, y) = $ "x and y are friends." Translate the proposition

   "Everyone who is smart has a friend who studies"

   into formal logic.

7. (6 points) Simplify $\neg\,(\forall x\ (x = 0 \lor \exists y\ xy \geq 1))$ to a logically equivalent proposition that does not use the negation symbol.

8. (8 points) Rewrite the proposition $p \leftrightarrow q$ using only the connectives $\neg, \land$. (That is, find another proposition that is logically equivalent to the given one, but uses only the symbols $(\,,\,), p, q, \neg, \land$.) Justify your answer.

9. (8 points) Prove, using the table of common valid arguments, that the following argument is valid. (It may help to rewrite some of the statements more formally first.)

Every student who gets an A on both the homework and the quiz did all the reading.	(a)
Olive is a student.	(b)
Olive did not do all the reading.	(c)
Olive got an A on the homework.	(d)

∴   Olive did not get an A on the quiz.

10. (8 points) Prove that if $7n - 3$ is even, then n is odd.

11. (6 points) Prove by contradiction: in every class of 30 students, there is some month in which at least three students have a birthday.

12. (8 points) Prove that for every integer n, $n^2 + 3n + 1$ is odd.

13. (8 points) Prove that for all sets A, B, $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

You may use the following theorems (we proved most of them in class), but indicate clearly where you use them. (You probably will not need to use all of them.)

1. For all sets X, Y, $X \cap Y \subseteq X$.
2. For all sets X, Y, $X \subseteq X \cup Y$.
3. If $X \subseteq Y$ and $X \subseteq Z$, then $X \subseteq Y \cap Z$.
4. If $X \subseteq Y \cap Z$, then $X \subseteq Y$.
5. If $X \subseteq Y$, then $\mathcal{P}(X) \subseteq \mathcal{P}(Y)$.

Scratch Paper - Do Not Remove

Scratch Paper - you may remove this if you find it convenient

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Logical equivalence(s)	Name
$p \vee \text{F} \equiv p$ $p \wedge \text{T} \equiv p$	Identity
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotence
$p \vee \text{T} \equiv \text{T}$ $p \wedge \text{F} \equiv \text{F}$	Domination
$\neg\neg p \equiv p$	Double-negative
$p \vee \neg p \equiv \text{T} \qquad p \wedge \neg p \equiv \text{F}$ $\neg \text{T} \equiv \text{F} \qquad \neg \text{F} \equiv \text{T}$	Complement
$p \vee (q \vee r) \equiv (p \vee q) \vee r$ $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$	Associativity
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutativity
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributivity
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption
$\neg(p \vee q) \equiv \neg p \wedge \neg q$ $\neg(p \wedge q) \equiv \neg p \vee \neg q$	DeMorgan
$p \to q \equiv \neg p \vee q$ $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$	Conditional Laws

Argument form	Argument Name
$p \to q$ p $\therefore \quad q$	Modus Ponens
$p \to q$ $\neg q$ $\therefore \quad \neg p$	Modus Tollens
$p \vee q$ $\neg p$ $\therefore \quad q$	Disjunctive Syllogism
$p \to q$ $q \to r$ $\therefore \quad p \to r$	Hypothetical Syllogism
$p \wedge q$ $\therefore \quad p$	Simplification
p $\therefore \quad p \vee q$	Addition
p q $\therefore \quad p \wedge q$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore \quad q \vee r$	Resolution

Argument form	Argument Name
c is an element (arbitrary or particular) $\forall x \; P(x)$ $\therefore \quad P(c)$	Universal Instantiation
c is an arbitrary element $P(c)$ $\therefore \quad \forall x \; P(x)$	Universal Generalization
$\exists x \; P(x)$ $\therefore \quad (c \text{ is a particular element}) \wedge P(c)$	Existential Instantiation
c is an element (arbitrary or particular) $P(c)$ $\therefore \quad \exists x \; P(x)$	Existential Generalization