

Name: \_\_\_\_\_

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have 75 minutes to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.

Question:	1	2	3	4	5	6	Total
Points:	12	15	32	10	6	10	85
Score:							

1. (12 points) Let  $\mathbf{u} = \langle 5, -2, 4, 7 \rangle$  and  $\mathbf{v} = \langle 1, 0, 3, -1 \rangle$ .

(a) Compute  $\mathbf{u} - 2\mathbf{v}$ .

(b) Compute  $\mathbf{u} \cdot \mathbf{v}$ .

(c) Compute  $\|7\mathbf{v}\|$ .

(d) Compute  $\text{proj}_{\mathbf{v}} \mathbf{u}$ .

2. Consider the line  $L$  parametrized by  $\ell(t) = \langle 1, 2, -1 \rangle + t\langle -3, 2, 1 \rangle$  and the point  $P = (5, -1, 1)$ .

(a) (3 points) Is  $P$  on  $L$ ?

(b) (8 points) Find the Cartesian equation of a plane containing both  $P$  and  $L$ .

(c) (4 points) Find the distance from  $P$  to  $L$ . (*No need to simplify.*)

3. Consider the vector equation  $\langle t, t^2, \frac{2}{3}t^3 \rangle$ . (*This problem continues onto the next page.*)

(a) (3 points) Find  $\mathbf{r}'(t)$ .

(b) (3 points) Find  $\mathbf{r}''(t)$ .

(c) (3 points) Find (and simplify)  $\|\mathbf{r}'(t)\|$ .

(d) (8 points) Find an equation for the tangent line to the curve at  $t = 1$ .

(e) (5 points) Find the arc length of the part of the curve with  $0 \leq t \leq 2$ .

*This is a continuation of the previous problem.*

(f) (6 points) Compute  $\hat{\mathbf{T}}(1)$ ,  $\hat{\mathbf{N}}(1)$ , and  $\hat{\mathbf{B}}(1)$ .

(g) (4 points) Compute  $\kappa(1)$ .

4. (10 points) Consider the two surfaces with equations  $z = 3y^2$  and  $x^2 + y^2 + z = 1$ . Let  $C$  be the curve that is their intersection. Find a parametrization of  $C$ , and sketch the surfaces together with  $C$ . (*Hint: eliminate  $z$  first, then choose  $x$  and  $y$ .*)

5. (6 points) Below is shown a trajectory of a particle (in the plane) together with the velocity at each of several points. Which direction is the acceleration at  $P$ ? Give brief justifications.  
(circle one answer in each row)

(a) in the page                      out of the page toward you                      into the page away from you

(b) vertical                                      to the left                                      to the right

(c) horizontal                                      upward                                      downward

6. (10 points) Circle 'True' or 'False' and give a brief justification.

(a) True      False      For every scalar function  $f(t)$  and every vector function  $\mathbf{r}(t)$ , we have  $\frac{d}{dt}(f(t)\mathbf{r}(t)) = f(t)\mathbf{r}'(t)$ .

(b) True      False      For a curve in the plane of the paper,  $\hat{\mathbf{B}}$  always points out of the page toward you (when it exists).

(c) True      False      Consider a (finite) curve  $C$  in  $\mathbb{R}^3$  and its projection/shadow  $P$  in the  $xy$ -plane. The arc length of  $C$  must be at least as large as the arc length of  $P$ .

(d) True      False      Consider a (finite) curve  $C$  in  $\mathbb{R}^3$  and its projection/shadow  $P$  in the  $xy$ -plane. The curvature of  $C$  (at a point  $(x, y, z)$ ) must be at least as large as the curvature of  $P$  (at the corresponding point  $(x, y, 0)$ ).



**Scratch Paper** - you may remove this if you find it convenient

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