Name: ______________________________

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have 75 minutes to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.

1. Rewrite the following as one double integral, then evaluate.

$$\int_{-2}^{0}\int_{-y}^{2} xy\ dx\ dy + \int_{0}^{2}\int_{0}^{2} xy\ dx\ dy + \int_{2}^{4}\int_{0}^{4-y} xy\ dx\ dy$$

2. Rewrite the following as one double integral, then evaluate.

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\ dy\ dx + \int_{1}^{\sqrt{2}}\int_{0}^{x} xy\ dy\ dx + \int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xy\ dy\ dx$$

3. Compute $\iint_R x^2 \, dA$, where R is the region bounded by $y = x^3 - x$ and $y = 0$. *(Caution: the region has two parts!)*

4. Compute $\iint_R x + y^2 \; dA$, where R is the region in the first quadrant bounded by $y = 1/x$, $y = 4/x$, $y = x$, and $y = 3x$.

5. Let D be the diamond $|x| + |y| \leq 1$.

   (a) Evaluate $\iint_D (2 + x^2y^3 - y^2 \sin x)\ dA$.

   (b) Evaluate $\iint_D e^{x+y}\ dA$.

6. Evaluate $\iint_R ye^{xy}\, dA$, where $R = [0,2] \times [0,3]$.

7. Compute the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, where a, b, c are positive constants. *(Hint: make an appropriate change of variables.)*

8. Find the volume inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 9$. Set up the integral for both cylindrical coordinates and spherical coordinates.

9. Find $\iiint_E x^2 \, dV$, where E is the region outside the cone $z = \sqrt{3}\sqrt{x^2 + y^2}$, inside the cone $z = \sqrt{x^2 + y^2}$, and below the plane $z = 2$. Set up the integral for both cylindrical coordinates and spherical coordinates.

10. Rewrite the following integral as an equivalent iterated integral in the five other orders.

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z)\ dy\ dz\ dx$$

11. Find the average distance from a point inside a ball of radius 1 to its center.

12. Find $\iiint_T xz\, dV$, where T is the solid tetrahedron with vertices $(0,0,0)$, $(\frac{1}{3},0,0)$, $(0,1,0)$, and $(0,0,1)$.

13. Find $\iiint_E z\, dV$, where E is the region inside the cone $z = \sqrt{x^2+y^2}$, inside the cylinder $x^2+y^2=1$, and below the plane $z=2$.

14. Change the order of integration to evaluate $\int_{-2}^{2}\int_{x^2}^{4}\int_{-\sqrt{y-x^2}}^{\sqrt{y-x^2}} \sqrt{x^2+z^2}\ dz\ dy\ dx$.

15. Use the transformation $x = \sqrt{2}u - \sqrt{2/3}v$, $y = \sqrt{2}u + \sqrt{2/3}v$ to evaluate the integral $\iint_R (x^2 - xy + y^2)\, dA$, where R is the elliptic disk $x^2 - xy + y^2 \leq 2$. *(Note: the axes of the ellipse are not parallel to the $x-$ and $y-$ axes, which is why we use the transformation.)*

16. True or false?

(a) If f is continuous on $[0,1]$, then $\int_0^1 \int_0^1 f(x)f(y)\ dx\ dy = \left(\int_0^1 f(x)\ dx\right)^2$.

(b) The integral $\int_0^{2\pi} \int_0^2 \int_r^2 dz\ dr\ d\theta$ represents the volume of the region inside the cone $z = \sqrt{x^2+y^2}$ and below the plane $z = 2$.

(c) Every triple integral is positive because it measures mass of a solid with density given by the integrand.

(d) If D is the disk $x^2+y^2 \leq 4$, then $\iint_D \sqrt{4-x^2-y^2}\ dA = \frac{16}{3}\pi$ because the integral measures the volume of a sphere of radius 2.