Name: ____________________

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have 120 minutes to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.

Here are some useful formulas. You are expected to know when they apply and what they mean.

$$\int_C \nabla f \; d\vec{r} = f(B) - f(A)$$

$$\iint_D (Q_x - P_y)\, dA = \oint_C P\, dx + Q\, dy$$

$$\iint_\Sigma \operatorname{curl} \vec{F} \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r}$$

$$\iiint_E \operatorname{div} \vec{F} \; dV = \iint_\Sigma \vec{F} \cdot d\vec{S}$$

$$\kappa = \frac{dT}{ds} = \frac{|r' \times r''|}{|r'|^3}$$

$$\cos^2 t = \frac{1}{2}(1 + \cos(2t))$$

$$\sin^2 t = \frac{1}{2}(1 - \cos(2t))$$

Question:	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Points:	10	3	3	8	10	10	10	12	8	8	12	25	3	122
Score:														

1. (10 points) Circle True or False. No partial credit, and no need to show work. Assume all functions have continuous derivatives.
   (a) True False If acceleration is constant, then motion is along a straight line.
   (b) True False If motion is along a straight line, then acceleration is constant.
   (c) True False A straight line has curvature 0 everywhere, regardless of the parametrization.
   (d) True False If curvature is 0 everywhere, then the curve is a straight line.
   (e) True False The definition for a function $f(x, y)$ to be *differentiable* is that its partial derivatives exist.
   (f) True False The Extreme Value Theorem says that if f is continuous on a region D that is closed and simply connected, then f attains a maximum and a minimum on D.
   (g) True False If $f > 0$ everywhere, then for any C, we have $\int_C f\,ds > 0$.
   (h) True False If $P, Q, R > 0$ everywhere, then for any C, we have $\int_C P\,dx + Q\,dy + R\,dz > 0$.
   (i) True False If $\vec{F}(x, y, z)$ is conservative, then $\operatorname{curl} \vec{F} = \vec{0}$ on the domain of $\vec{F}$.
   (j) True False If $\operatorname{curl} \vec{F} = \vec{0}$ on the domain of $\vec{F}$, then $\vec{F}$ is conservative.

2. (3 points) Simplify $3\langle 2, 1, 4\rangle - \langle -1, 2, 5\rangle$.

3. (3 points) Compute $\operatorname{div} \langle x^4y^2, \sin(xy)\rangle$.

4. (8 points) A particle's position at time t is given by $(2t^2 - 3, 2t, e^t)$. Find the velocity, speed, and acceleration of the particle at time $t = 0$.

5. (10 points) Evaluate $\iint_R \sin(x^2 + y^2)\; dA$, where R is the region in the first quadrant bounded by the y-axis, the line $y = x$, and the circle $x^2 + y^2 = 9$.

6. (10 points) Use Lagrange multipliers to find the minimum and maximum values of $f(x, y) = x^2 + y^2$ subject to the constraint $xy = 1$. If either doesn't exist, explain why.

7. (10 points) Let C be the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = y + 2$, and let $\vec{F} = \langle 2y,\, xz,\, x + y \rangle$. Use Stokes's Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is oriented counterclockwise when viewed from above. Make sure to check for correct orientations.

8. (12 points) Consider $\vec{F}(x,y) = \langle ye^{xy} + 2x,\, \sin(y) + xe^{xy} \rangle$.

   (a) Find a potential function for $\vec{F}$.

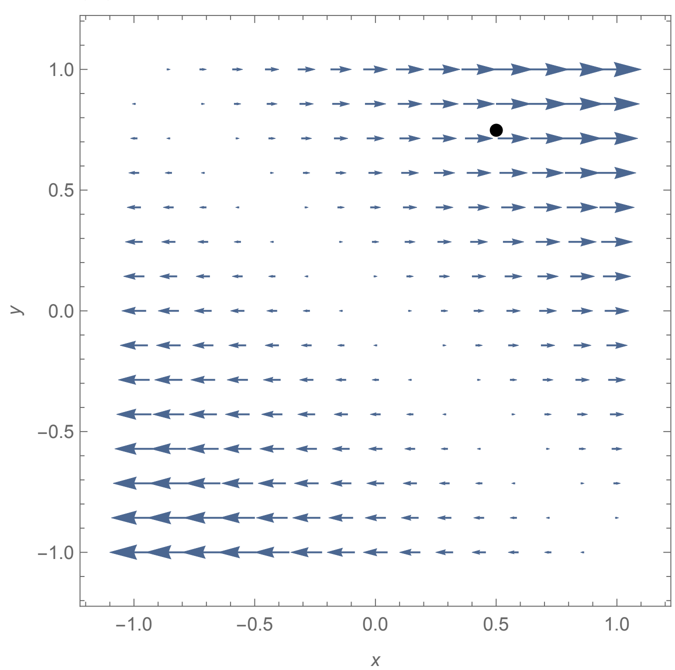
   (b) Compute $\displaystyle\int_C \vec{F} \cdot d\vec{r}$, where C is the curve given by $\left\langle 2\sin(t),\, te^{\cos t} \right\rangle$, $t \in [0, \pi/2]$.

9. (8 points) Set up the integral $\int_C x^2y\ ds$, where C is the curve with parametrization $\langle \cos t, \sin t, t^2\rangle$, $t \in [0, \pi/2]$. Stop when you have an ordinary Calculus 1 integral (with one variable and no vectors).

10. (8 points) Find the limit if it exists. Justify your answer.

$$\lim_{(x,y)\to(0,0)} \frac{x^3y - xy^3}{x^4 + 2y^4}$$

11. (12 points) Below is shown a plot of $\vec{F}$ and a point P. Justify your answers.

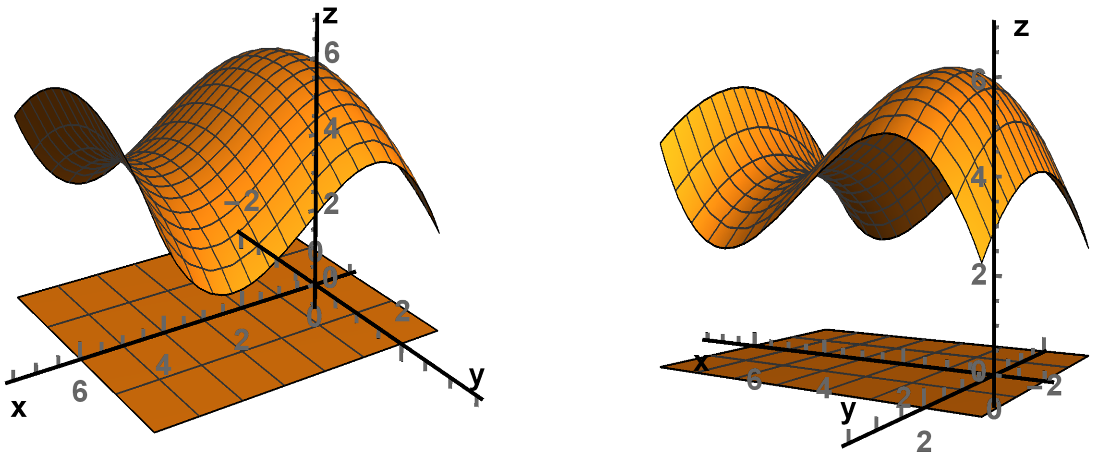


(a) Which direction is $\operatorname{curl} \vec{F}$ at P? (Assume that $\vec{F}$ is the same in each plane parallel to the page.)

(b) What is the sign of $\operatorname{div} \vec{F}$ at P?

(c) Is $\vec{F}$ conservative?

12. Below is shown the graph of a function $f(x,y)$ over the solid rectangle R with $-1 \le x \le 6$, $-2 \le y \le 2$, $z = 0$. When asked to estimate, use exact formulas where possible then give estimates for the expressions in those formulas. If you use a theorem, say which one.



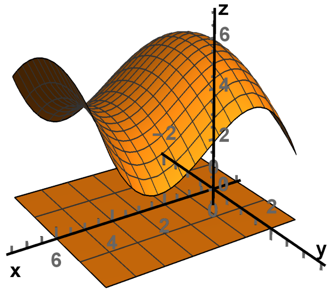
(a) (5 points) Locate (approximately) and classify the critical points of f on the interior of R.

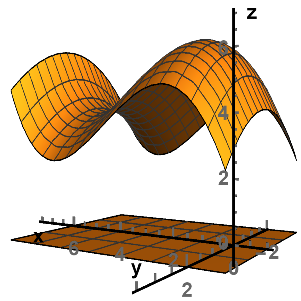
(b) (5 points) Compute/estimate $\oint_{C_2} \nabla f \cdot \vec{dr}$, where C_2 is the boundary of R.

(c) (5 points) Let Σ be the graph of f over R. Compute/estimate $\iint_\Sigma y\, dS$.

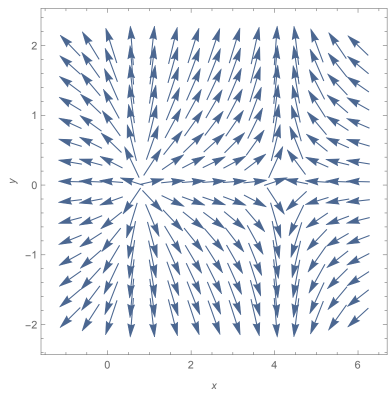
(d) (5 points) Compute/estimate the average value of f on R.

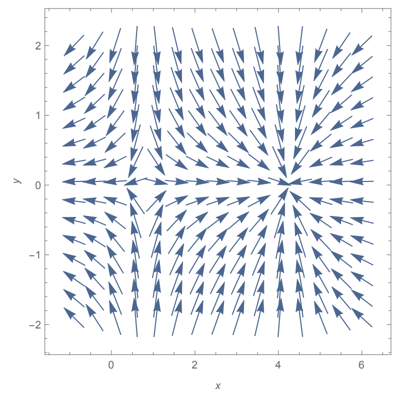
(This is a continuation of number 12. The images are repeated here for your convenience.)

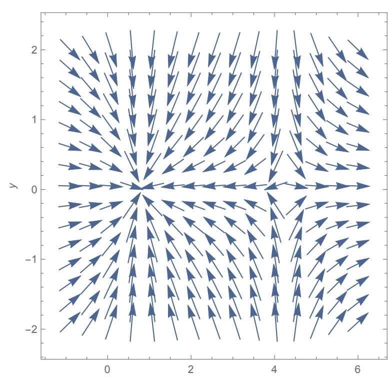


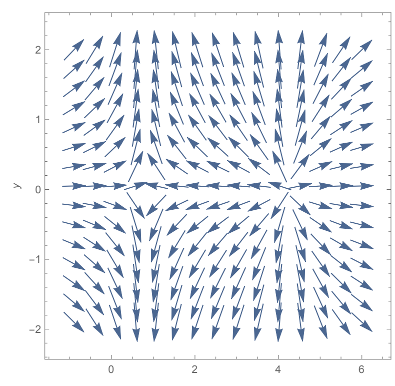


(e) (5 points) Which of the following images is the plot of ∇f?









(f) (5 points (bonus)) Suppose $Q(x, y)$ is such that $Q_x = f$. Compute/estimate $\oint_{C_2} \langle x^2, Q\rangle \cdot \vec{dr}$, where C_2 is the boundary of R.

13. Consider the lines

$$\ell_1(t) = (3,1,1) + t(6,-2,4) \qquad \text{and}$$
$$\ell_2(t) = (5,3,6) + t(-9,3,-6).$$

(a) (3 points) Are the lines parallel or not? How do you know?

(b) (7 points (bonus)) Find the distance between the lines.

Scratch Paper - Do Not Remove

If you need additional scratch paper, please ask a proctor.

Scratch Paper - you may remove this if you find it convenient

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