

Name: \_\_\_\_\_

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have 75 minutes to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.

You may or may not need the following formulas:

$$\sin^2 t = \frac{1}{2}(1 - \cos(2t))$$

$$\cos^2 t = \frac{1}{2}(1 + \cos(2t))$$

Question:	1	2	3	4	5	6	7	8	Total
Points:	10	6	8	2	7	10	6	3	52
Score:									

1. (10 points) Find the volume of the region bounded by  $z = y$ ,  $x = y^2$ ,  $x = 1$ , and  $z = 1$ .

2. A metal sheet occupies the rectangle  $[0, 7] \times [0, 9]$ . It has non-uniform density; you are given the densities at various points:

	9	1	2	4	6	5	3	1	2
	8	0	1	3	4	5	3	2	1
	7	5	3	5	7	6	4	2	1
	6	2	4	5	6	7	4	3	2
	5	3	5	6	8	9	6	5	3
$y$	4	1	4	6	7	9	6	4	3
	3	2	3	4	6	7	5	4	3
	2	4	5	3	4	5	3	2	1
	1	5	6	4	3	3	1	1	2
	0	6	8	7	4	2	1	2	3
$\rho(x, y)$		0	1	2	3	4	5	6	7
		$x$							

- (a) (3 points) Estimate the mass of the part of the sheet  $R$  with  $1 \leq x \leq 5$  and  $4 \leq y \leq 8$ , using a Riemann sum with two subintervals in each direction (the book/WebAssign would say  $m = n = 2$ ).

- (b) (3 points) Estimate  $\iint_R x\rho(x, y) \, dA$ , using again a Riemann sum with two subintervals in each direction.

3. (8 points) Set up the integral in spherical coordinates (*you do not have to evaluate it*):

$$\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_0^{\sqrt{9-x^2-y^2}} \sqrt{x^2+y^2} \, dz \, dx \, dy$$

4. (2 points) Circle ‘True’ or ‘False’ (1 point each):

(a) True      False      For any two functions  $f, g$  of three variables and any region  $E$ ,

$$\iiint_E (f + g) \, dV = \iiint_E f \, dV + \iiint_E g \, dV.$$

(b) True      False      For any two functions  $f(x, y)$  and  $g(z)$  and any region  $E$ , if  $I$  is the shadow of  $E$  on the  $z$ -axis and  $D$  the shadow in the  $xy$ -plane, then

$$\iiint_E f(x, y)g(z) \, dV = \left( \iint_D f(x, y) \, dA \right) \left( \int_I g(z) \, dz \right)$$

5. (7 points) Evaluate  $\iint_R \frac{3x-y}{x+3y} dA$ , where  $R$  is the parallelogram enclosed by the lines  $3x - y = 2$ ,  $3x - y = 4$ ,  $x + 3y = 5$ , and  $x + 3y = 7$ .

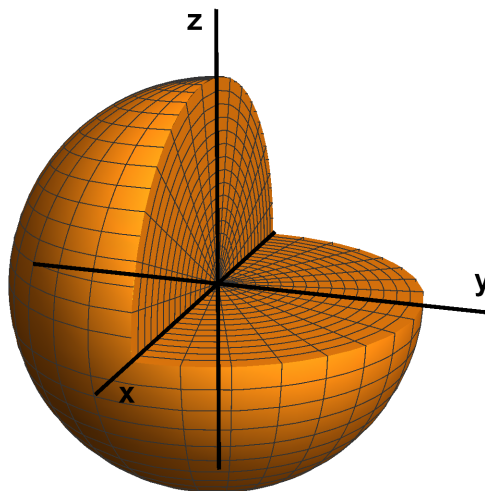
6. (10 points) Compute  $\int_1^6 \int_0^{\ln x} y \, dy \, dx$  by changing the order of integration.

7. (6 points) Below is a region  $E$ . For each part, circle the sign of the integral and give brief justification:

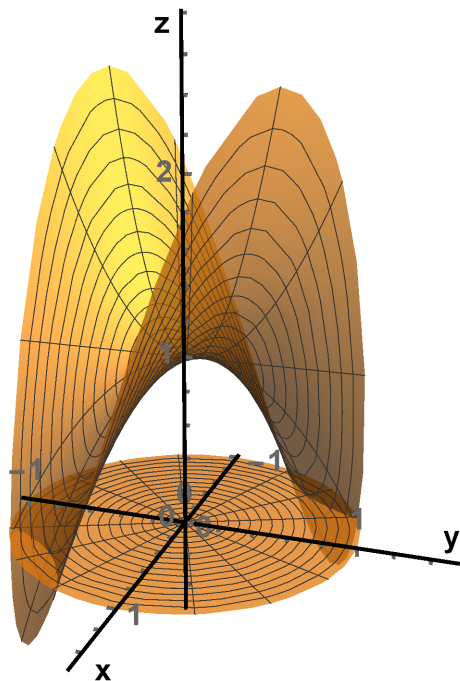
$$\iiint_E xy \, dV \quad + \quad 0 \quad -$$

$$\iiint_E y \, dV \quad + \quad 0 \quad -$$

$$\iiint_E \left( z + \frac{1}{2} \right) dV \quad + \quad 0 \quad -$$



8. (3 points) Below is shown the graph of a function  $f$  over the unit disk  $R$  in the  $xy$ -plane. Estimate the average value of  $f$  on  $R$ .



**Scratch Paper - Do Not Remove**



**Scratch Paper** - you may remove this if you find it convenient

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